I

a) $\lim_{x\to 0^+}\left[x^{\left(\frac{\ln a}{1+\ln x}\right)}\right]$

$\lim_{x\to 0^+}\ln Y = \frac{\ln a}{1+\ln x}\ln x \quad \frac{-\infty}{-\infty}$

$\lim_{x\to 0^+}\ln Y = \frac{\frac{\ln a}{x}}{\frac{1}{x}} = \ln a$

$\lim_{x\to 0^+}\ln a = \ln a$

$\ln y = \ln a \therefore y = a.$

b) $\lim_{x\to\infty}\left[x^{\frac{\ln a}{1+\ln x}}\right]$

$\lim_{x\to\infty}\ln Y = \frac{\ln a}{1+\ln x}\cdot\ln x \quad \frac{\infty}{\infty}$

Sameas para.

$\lim_{x\to\infty}\ln a = \ln a$

$\ln Y = \ln a \therefore Y = a.$

c) $\lim_{x\to 0}\left[(x+1)^{\frac{\ln a}{x}}\right]$

$\lim_{x\to 0}\ln Y = \frac{\ln a}{x}\ln(x+1) \quad \frac{0}{0}$

$= \frac{\ln a}{\frac{x+1}{1}}$

$\lim_{x\to 0}\frac{\ln a}{x+1} = \ln a.$

$\ln y = \ln a \therefore y = a$

Again!

d) $\lim_{x\to\infty}\frac{\frac{\pi}{2}-\tan^{-1}x}{\frac{1}{x}} \quad \frac{0}{0}$

$\lim_{x\to\infty}\frac{\frac{-1}{1+x^2}}{\frac{-1}{x^2}}$

$\lim_{x\to\infty}\frac{x^2}{1+x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty}\frac{1}{\frac{1}{x^2}+1} = 1$

e) $\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\sin\frac{1}{x}} \quad \frac{0}{0}$

$\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}}{\cos\frac{1}{x}\cdot\frac{-1}{x^2}}$

$\frac{1}{1+0} = \frac{1}{1} = 1$

f) $\lim_{x\to 0}\frac{\tan^{-1}x}{\tan^{-1}2x} \quad \frac{0}{0}$

$\lim_{x\to 0}\frac{\frac{1}{1+x^2}}{\frac{2}{1+4x^2}} = \frac{1}{2}$

g) $\lim_{x\to 0}(1+\sin 4x)^{\cot x}$

$\lim_{x\to 0}\ln Y = \cot x\ln(1+\sin 4x) = \frac{\ln(1+\sin 4x)}{\tan x} \quad \frac{0}{0}$

$\lim_{x\to 0}\frac{\frac{4\cos 4x}{1+\sin 4x}}{\sec^2 x} = \frac{4}{1} = 4$

$\ln y = 4$

$y = e^4$

II $f(x) = 4x^3 - 1$

no 2nd fct'n → $g(x) = 4x^3 - 1 - x$

$g(\frac{1}{2}) = \frac{1}{2} - 1 - \frac{1}{2} = -1$

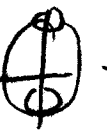
$g(1) = 4 - 1 - 1 = 2$

By IVT there exists x such that $g(x) = 0$

∴ there is a fixed point so that $4x^3 - 1 = x$.

III $f(x) = \ln(2 + \sin x) \quad 0 \le x \le 2\pi$

$f'(x) = \frac{\cos x}{2 + \sin x} \overset{\text{set}}{=} 0$



f' : + 0 − 0 + (0, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, 2π)

$(0, \ln 2)$ min. END PT INC fct.

$(\frac{\pi}{2}, \ln 3)$ max f'

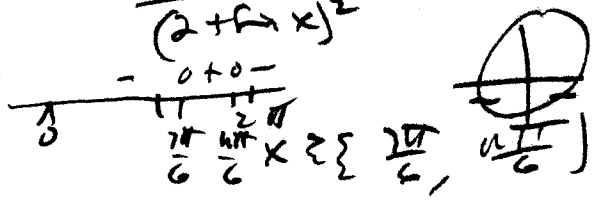
$(\frac{3\pi}{2}, 0)$ min f'

$(2\pi, \ln 2)$ max END PT INC to it.

$$f'' = \frac{-\sin x(2 + \sin x) - \cos^2 x}{(2 + \sin x)^2}$$

$$\frac{-2\sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2}$$

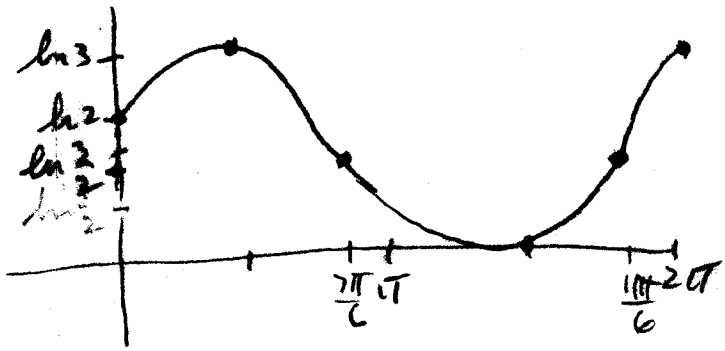
$$\frac{-2\sin x - 1}{(2 + \sin x)^2} \overset{\text{set}}{=} 0 \quad \sin x = -\frac{1}{2}$$



f'' : − 0 + 0 − (0, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$, 2π)

$x \in \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$

PI $(\frac{7\pi}{6}, \ln\frac{3}{2})$ $(\frac{11\pi}{6}, \ln\frac{3}{2})$



IV $y = x^3 + ax^2 + bx + 1 \quad [0, 2]$

$y|_{x=1} = 6 = 1 + a + b + 1$

$a + b = 4$

$y' = 3x^2 + 2ax + b$

$y''|_{x=1} = 6x + 2a = 6 + 2a \overset{\text{set}}{=} 0$

$a = -3$

$\therefore b = 7$

$y = x^3 - 3x^2 + 7x + 1$

$y' = 3x^2 - 6x + 7 \overset{\text{set}}{=} 0$

$x = \frac{6 \pm \sqrt{36 - 84}}{6}$ Reject.

y' always > 0

y is strictly increasing.

$y|_{x=0} = 1 \quad y|_{x=2} = 8 + 12 + 14 + 1 = 11$

min $(0, 1)$ END PT INC fct.

max $(2, 11)$ END PT INC to it.

Extra. $\lim_{x \to 0} \frac{\cos(ax + b)}{ax^2} = -4$

for l'Hopital to work $\cos(ax)|_{x=0} = b$ and $a > 0 \Rightarrow b$

$b = 1$

then $\lim_{x \to 0} \frac{-a\sin ax}{4x} = \lim_{x \to 0} -\frac{a}{4} \cdot \frac{\sin ax}{ax} \cdot a \overset{\text{set}}{=} -4$

$-\frac{a^2}{4} = -4$

$a^2 = 16$

$a = \pm 4$.

So $a = \pm 4$

$b = 1$

12/19/2000

100	
90	2
80	5
70	9
60	4
59	5

100

$\frac{1871}{26}$ w/o the bottom 2, average goes up !! 3 pts..

720

This is an excellent exam. There is nothing confusing here about the mean. Possibly the II problem should be moved to the 5 section and be replaced by Law of the mean problem.

I liked this exam as it was the students were focused as they should have been — maybe this was a wake up call for them?

most students refused to listen to the "cookie fairy" so you can apply the EVT.

Lower Scores did the 1st 3 of part 7

$\ln y = \frac{\ln a}{1+\ln x} \cdot \ln x$ check ok.

$\frac{1}{y}\frac{dy}{dx} =$ etc.

I like the book the derivative instead of applying l'Hopital's rule only on the right side.

Advanced Placement Calculus

Name Calc Ch 6

Per _____ Date Mach 2000

Copy original problem.

Convince *me* that **you** understand the concept!

No calculators, of course.

Chapter 6 Mechanical Exam

I

Evaluate the following limits. Use l'hopital's rule at least one time. (Be very sure your methods are explicit.): (50 pts tot)

a) $\lim_{x\to 0^+}\left[x^{\left(\frac{\ln a}{1+\ln x}\right)}\right]$ b) $\lim_{x\to\infty}\left[x^{\left(\frac{\ln a}{1+\ln x}\right)}\right]$ c) $\lim_{x\to 0}\left[(x+1)^{\left(\frac{\ln a}{x}\right)}\right]$

d) $\lim_{x\to\infty}\dfrac{\frac{\pi}{2}-\tan^{-1}x}{\frac{1}{x}}$ e) $\lim_{x\to\infty}\dfrac{\ln\left(1+\frac{1}{x}\right)}{\sin\frac{1}{x}}$ f) $\lim_{x\to 0}\dfrac{\tan^{-1}x}{\tan^{-1}2x}$

g) $\lim_{x\to 0}(1+\sin 4x)^{\cot x}$

II

A function is said to have a *fixed point* on an interval (a, b) if there is some point $x = p$, $a < p < b$ such that $f(p) = p$. Show that $f(x) = 4x^3 - 1$ has a fixed point on the interval $\left(\frac{1}{2},1\right)$. (10 pts)

III

Given $f(x) = \ln(2 + \sin x)$ for $0 \le x \le 2\pi$. (25 pts tot)

a) Find and identify coordinates of all extrema.

b) Find coordinates of all points of inflection.

c) Sketch. Be sure all features are clearly labeled.

IV

Given $y = x^3 + ax^2 + bx + 1$ for $x \in [0,2]$. (15 pts tot)

a) Find the values of a and b so that (1, 6) is an inflection point.

b) Find and identify all extreme values.

Extra Credit ---------- 5 pts ----------

Find a and b so that: $\lim_{x\to 0}\dfrac{\cos(ax)-b}{2x^2} = -4$

I A) $\lim_{x\to 1} \frac{x^{\frac{1}{3}} - x^{\frac{1}{7}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$ $\frac{1-1}{1-1}$ $\frac{0}{0}$

$\lim_{x\to 1} \frac{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{7}x^{-\frac{6}{7}}}{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{5}x^{-\frac{4}{5}}}$

$\frac{\frac{1}{3} - \frac{1}{7}}{\frac{1}{3} - \frac{1}{5}}$ $\frac{7-3}{21}$ $\frac{5-3}{15}$ $\frac{4}{21}\cdot\frac{15}{2}$

$\frac{10}{7}$

b) $\lim_{x\to\infty} \frac{x^{\frac{1}{3}} - x^{\frac{1}{7}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$ $\frac{x^{\frac{1}{3}}}{x^{\frac{1}{7}}}$

$\lim_{x\to\infty} \frac{1 - \frac{1}{x^{\frac{2}{21}}}}{1 - \frac{1}{x^{\frac{2}{15}}}} = 1$

c) $\lim_{x\to 1} x^{\frac{1}{\ln x}}$ $\left\{\to \frac{1}{\log_e x} = \frac{1}{\frac{\log_x x}{\log_x e}}\right.$

$\lim_{x\to 1} x^{\log_x e} = e$

II a) $0 < \frac{a}{a+b} < 1$

$0 < \frac{a}{a} < 1$

$0 < \frac{a}{a+b} < \frac{a}{a}$ since $a+b > a$

$a < \frac{a}{a+b} < 1$

b) $f(x) = |x| = \sqrt{x^2}$

$f'(x) = \frac{1}{2}(x^2)^{-\frac{1}{2}} \cdot 2x$

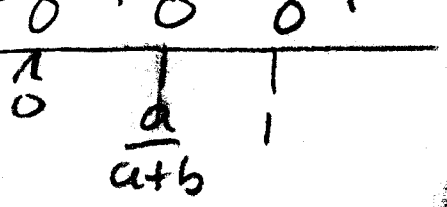
$= \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$

c) $g'(x) = |x|^{a-2}|x-1|^{b-2}x(x-1)(a+b)\left[x - \frac{a}{a+b}\right] \overset{\text{set}}{=} 0$

Never zero

we know $\frac{a}{a+b} < 1$ by part A.

①



Rel max at $x = \frac{a}{a+b}$ f' test.

② $g\left(\frac{a}{a+b}\right) = \left|\frac{a}{a+b}\right|^{a-2}\left|\frac{a}{a+b} - 1\right|^{b-2}\frac{a}{a+b}$

$\frac{b}{a+b} > 0 \therefore \frac{-b}{a+b} < 0$

Def of abs says $\left|\frac{-b}{a+b}\right|$ is $-\left(\frac{-b}{a+b}\right) = \frac{b}{a+b}$

$\frac{a^a}{(a+b)^a} \cdot \left(\frac{a-a-b}{a+b}\right)^b$

$\frac{a^a}{(a+b)^a} \cdot \frac{|(-b)|^b}{(a+b)^b}$

$\frac{a^a}{(a+b)^a} \cdot \frac{b^b}{(a+b)^b}$

Neg sign dropped Because of abs value

$\frac{a^a b^b}{(a+b)^{a+b}}$

III $f(x) = \frac{x}{(x-1)^2}$ u v

$f'(x) = \frac{(x-1)^2 - x\cdot 2(x-1)}{(x-1)^4}$

$\frac{x-1-2x}{(x-1)^3}$

$-\frac{(x+1)}{(x-1)^3}$ u v

f': − 0 + ∄ − ; at -1, 0

min occurs at $x=-1$ f' test.

$(-1, -\frac{1}{4})$ No max

$f'' = \frac{(x-1)^3(-1) + (x+1)3(x-1)^2}{(x-1)^6}$

$\frac{-(x-1)+3(x+1)}{(x-1)^4}$

$\frac{2x+4}{(x-1)^4}$

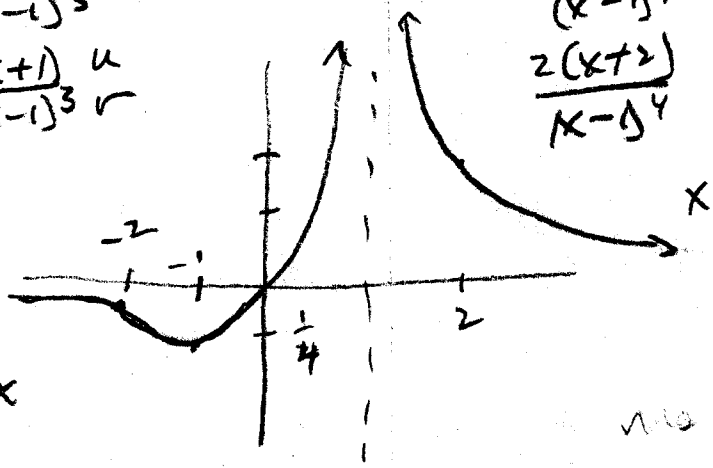
$\frac{2(x+2)}{(x-1)^4}$

f'': 0 + ∄ + ; at -2, 0, 1

$x=1$ is where der DNE so not PI

$x=-2$ is PI changes concavity.

$-\frac{2}{9}$ PI



IV $f(x) = x^x = y$

$\ln y = x \ln x$ u v

$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$

$\frac{dy}{dx} = x^x(\ln x + 1) \overset{\text{set}}{=} 0$

$x = e^{-1}$

min f' test

− 0 + ; at 0, $\frac{1}{e}$, 1

V $f(x) = \frac{\sin x}{x}$ on $0 < x < \frac{\pi}{2}$

WTP $f'(x) < 0$

$f'(x) = \frac{x\cos x - \sin x}{x^2} \overset{?}{<} 0$

Let $g(x) = x\cos x - \sin x$ u v

$g(0) = 0$

$g'(x) = \cos x - x\sin x - \cos x$

$= -x\sin x$

$- + + = - < 0$ } 5 pts

so g decreases from 0.

$g < 0 \therefore f' < 0 \therefore$

f is a decreasing fctn.

EXTRA.

$g(x) = |x|^a |x-1|^b$

$g'(x) = |x-1|^b a|x|^{a-1}\frac{d(|x|)}{dx} + |x|^a b|x-1|^{b-1}\frac{d(|x-1|)}{dx}$

$= |x-1|^b a|x|^{a-1}\cdot\frac{x}{|x|} + |x|^a b|x-1|^{b-1}\frac{(x-1)}{|x-1|}$

$= ax|x-1|^b|x|^{a-2} + b(x-1)|x|^a|x-1|^{b-2}$

$= |x|^{a-2}|x-1|^{b-2}\left[ax(x-1)^2 + b(x-1)x^2\right]$

$\to |x|^{a-2}|x-1|^{b-2}(x-1)[ax(x-1) + bx^2]$

$|x|^{a-2}|x-1|^{b-2}(x-1)[ax^2 - ax + bx^2]$

$|x|^{a-2}|x-1|^{b-2}(x-1)x[ax - a + bx]$

$|x|^{a-2}|x-1|^{b-2}x(x-1)[x(a+b) - a]$

$|x|^{a-2}|x-1|^{b-2}x(x-1)(a+b)\left[x - \frac{a}{a+b}\right]$

100	0
90	2
80	4
70	4
60	3
59	3

94

5

1158

6

Great exam!

No changes.

30 se Students. 72.4

11/19/96

100	2
90	5
80	5
70	1
60	3
59	7

Jury duty

75.9

11/19/01

100	0
90	4
80	8
70	5
60	2
59	5

95

43

1786

24

74.4

The bottom ones are entrenched there.

Argh!!

Advanced Placement Calculus Name Calc Ch 6 mech

Copy original problem. Per _____ Date 12/1/94

Convince *me* that you understand the concept!

11/19/96
12/19/01

Chapter 6 Mechanical Exam

I Find the following limits. (All limits do exist.) Be very sure your reasoning is clear. (5 pts ea)

a) $\lim_{x \to 1} \frac{x^{1/3} - x^{1/4}}{x^{1/3} - x^{1/5}}$ b) $\lim_{x \to \infty} \frac{x^{1/3} - x^{1/4}}{x^{1/3} - x^{1/5}}$ c) $\lim_{x \to 1} x^{\frac{1}{1-x}}$

II Given the real numbers a and $b > 0$ (40 pts tot)

a) Argue that: $0 < \frac{a}{a+b} < 1$

b) Given $f(x) = |x|$. Using the fact that $f(x) = \sqrt{x^2}$, show that $f'(x) = \frac{x}{|x|}$.

c) Given $g(x) = |x|^a |x-1|^b$ and $g'(x) = |x|^{a-2} |x-1|^{b-2} x(x-1)(a+b)\left[x - \frac{a}{a+b}\right]$

1) For which x does g have a relative maximum?

2) Show that this relative maximum value is $\frac{a^a b^b}{(a+b)^{a+b}}$

III Given $f(x) = \frac{x}{(x-1)^2}$. Determine extrema, state coordinates of points of inflection and draw the graph of the function showing these points. (20 pts)

IV For which $x > 0$ does $f(x) = x^x$ achieve its minimum value? (10 pts)

V Prove that $f(x) = \frac{\sin x}{x}$ is a decreasing function for $0 < x < \pi/2$ (15 pts)

Extra Credit ---------- 5 pts ----------

Show that the derivative of $g(x)$ in part II C is as it is stated.

I $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ $\frac{\infty}{\infty}$ Can't use l'hopital's rule because it goes in a circle.

$\lim_{x\to\infty} \frac{1}{\frac{2x}{2\sqrt{x^2+1}}}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$

$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} \quad \frac{\frac{1}{x}}{\frac{1}{x}}$

$\lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = 1$

II $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$

let $y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x \ln\left(1+\frac{1}{x}\right)$

$\lim_{x\to\infty} = \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ $\frac{0}{0}$

$\lim_{x\to\infty} \left(\ln y = \frac{\frac{1}{1+\frac{1}{x}}\left(\frac{-1}{x^2}\right)}{\frac{-1}{x^2}}\right)$

$= \frac{1}{1+\frac{1}{x}} = 1$

$\lim_{x\to\infty} \ln y = 1$

$\ln y = 1$

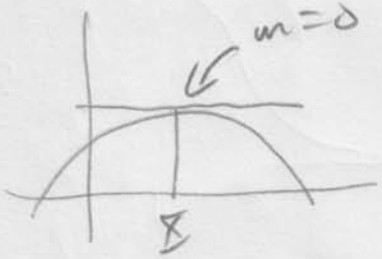
$y = e^1 = e$

III

A Rolle's Thm

if $f(a) = f(b)$, f cont in $[a,b]$

$\exists\ \bar{x} \in (a,b) \Rightarrow f'(\bar{x}) = 0$

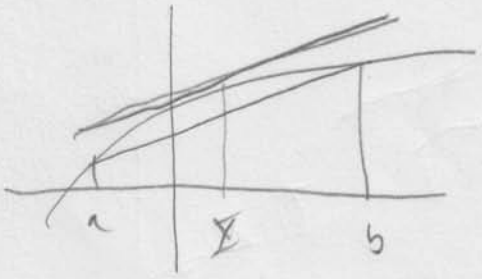


B) mean value thm } something.

C) rawdog mean

f is cont in $[a,b]$, f' exists in (a,b) $\exists\ \bar{x} \Rightarrow f'(\bar{x}) = \frac{f(b)-f(a)}{b-a}$

it tangent line // to chord.



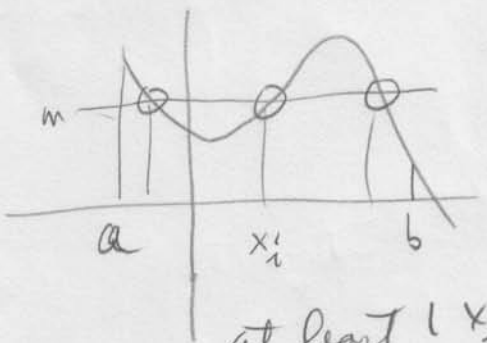
D) Intermediate Value Thm

if f is cont in $[a,b]$ of

$f(a) \le m \ge f(b)$ (or vice versa)

then $\exists$ at least one

$\bar{x} \Rightarrow f(\bar{x}) = m$.



at least 1 x_i guaranteed by IVT

IV $y = \left(\frac{2^x + 8^x}{2}\right)^{\frac{1}{x}}$ $\quad 2^{3x} = 2^x \cdot 2^{2x}$

a) $\ln y = \frac{1}{x}\ln(2^x + 8^x) - \ln 2$

$= \frac{\ln 2^x(1+4^x) - \ln 2}{x}$

$\ln y = \frac{\ln 2^x + \ln(1+4^x) - \ln 2}{x}$

b) $\lim_{x\to 0} \ln y = \frac{0 + \ln 2 - \ln 2}{0} \quad \frac{0}{0}$

$\lim_{x\to 0} \ln 2 + \frac{4^x \ln 4}{1+4^x}$

$\ln 2 + \frac{2\ln 2}{2}$

$\ln y = 2\ln 2 = \ln 4$

$\therefore y = 4$

V

a) xe^{-x}

$f'(x) = (1-x)e^{-x}$ $\quad f'$: + 0 − (0, 1)

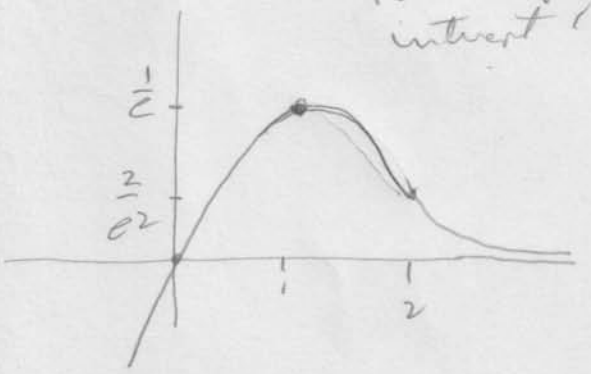
$f''(x) = (x-2)e^{-x}$ $\quad f''$: − 0+ (0, 2)

$f(1) = \frac{1}{e}$ $\quad (1, \frac{1}{e})$ max ~~f'~~

$f(2) = \frac{2}{e^2}$ PI $\quad \lim_{x\to 0} f(x) \to 0^+$

$y = 0$ asymptotes

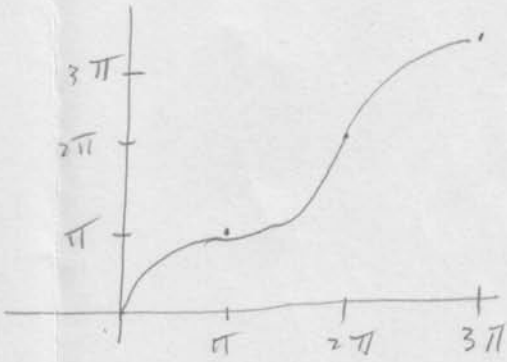
intercept $(0,0)$



b) $f(x) = x + \sin x$

$f'(x) = 1 + \cos x$ $\quad f'$: + ⊗ + ⊗ + (0, π, 2π, 3π)

$f''(x) = -\sin x$ $\quad f''$: − 0 + 0 − 0 (0, π, 2π, 3π)



CPI (π, π)

4 stat PI $(2\pi, 2\pi)$

min $(0,0)$ INC

first.

max $(3\pi, 3\pi)$ INC

& 1st.

only intercept $(0,0)$

12/1/99

96 Renata Bradsky
90 Zac Hall
82 Sarah Sutton
82 Jon Eustis
73 Rajesh Basra
65 Melissa Vandertolen
63 Andrea Marlow
55 Jim Garvey

Good exam.
No changes

11/28/01

93 Jimmy Alonso
80 Stacey Cox
80 Cara Alexander
77 Rachel Cunningham
76 Annie Gutierrez
75 Ashley Carter
50 Brittany Gonzales

Advanced Placement Calculus

Name Calc

Per _____ Date CC mech

Copy original problem.

Convince *me* that **you** understand the concept*!*

No Calculators!

MAKE up.
99/01

Chapter 6 Mechanical Make-Up Exam

I Given: $\lim_{x \to \infty} \frac{x}{\sqrt{x^2 + 1}}$.

Why is L'Hopital's rule of no value with this problem?
What is the limit? (10 pts)

II Use L'Hopital's rule to show that $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$ (10 pts)

III Explain-um's: Define each of the following. For each, draw a sketch illustrating your definition. Be sure to indicate on your picture all points mentioned. Do NOT use specific numbers. (20 pts tot)

a) Rolle's Theorem

b) Mean Value Theorem

c) Law of the Mean

d) Intermediate Value Theorem

IV Given: $y = \left(\frac{2^x + 8^x}{2}\right)^{\frac{1}{x}}$. (20 pts tot)

a) Show that $\ln y = \frac{\left(\ln 2^x + \ln(1 + 4^x) - \ln 2\right)}{x}$

b) Find $\lim_{x \to 0} y$

V For each of the following: Determine the coordinates of all max, min, PI, and intercepts. Determine the equation(s) of any asymptotes. Sketch $f(x)$ illustrating all significant points.
Include appropriate number lines, of course. (20 pts ea)

a) $f(x) = xe^{-x}$

b) $f(x) = x + \sin x \quad 0 \le x \le 3\pi$

I $f(x) = 2\sin^{-1}x$

a) $f(0) = 2\sin^{-1}0 = 0$

$f(1) = 2\sin^{-1}1 = 2\cdot\frac{\pi}{2} = \pi$

$= $

$f'(x) = \frac{2}{\sqrt{1-x^2}} \overset{\text{set}}{=} \frac{\pi - 0}{1-0}$

$\frac{2}{\sqrt{1-\frac{\pi^2-4}{\pi^2}}} = \frac{2}{\frac{\sqrt{\pi^2-\pi^2+4}}{\pi}}$ $\quad \frac{4}{\pi^2} = 1-x^2$

$= \frac{2}{\frac{2}{\pi}} = \pi$ $\quad x = \pm\sqrt{1-\frac{4}{\pi^2}}$

$x > 0 \therefore x = \frac{\sqrt{\pi^2-4}}{\pi} \triangleq .771$

b) $g(x) = \cos^{-1}(1-2x^2)$

$g(0) = \cos^{-1}(1) = 0$

$g(1) = \cos^{-1}(1-2)$

$= \cos^{-1}(-1) = \pi$

$g'(x) = \frac{-(-4x)}{\sqrt{(1-(2x^2)^2}}$

$\frac{+4x}{\sqrt{1-1+4x^2-4x^4}}$

$\frac{4x}{\sqrt{4x^2-4x^4}}$

$\frac{4x}{2x\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$

$\#x^2(1-x^2)$

c) $x = .771$ and $f'(c) = 3.142$

d) Note $f' = g'$. $f(0) = g(0)$

$f(1) = g(1)$

e) if 2 fcts have the same der, they differ by a constant. These don't even differ by a constant. conclusion: $f(x) = g(x)$.

II f' $+ \; 0 \; - \; 0 \; + \; \otimes \; +$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

f'' $- \; 0 \; + \; 0 \; - \; 0 \; +$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

a) $x = -3$ min inc fn'l

$x = -2$ max f' cont

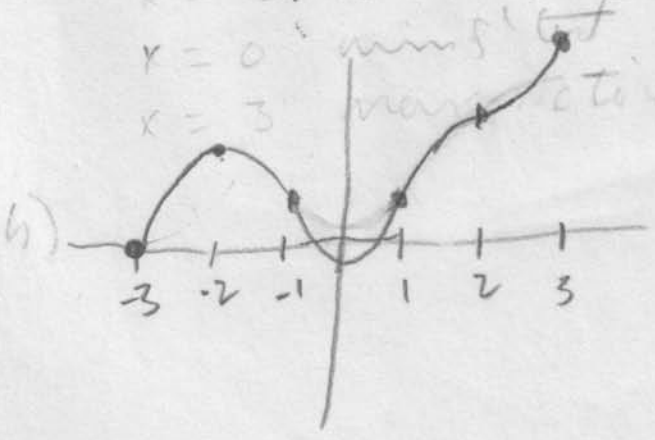
$x = 0$ min f' cont

$x = 3$ max inc fn'l.

b) $x = -1$ concave up

$x = 1$ $-1 < x < 1$ or

$x = 2$ $2 < x \leq 3$



III WTP $\frac{x}{1+x^2} < \tan^{-1}x < x \quad x>0$

$f \qquad g \qquad h$

$f(0) = g(0) = h(0) = 0$

If $f' < g' < h'$ Then $\forall x > 0$

f increases slower than g which increases slower than h Thus f will be less than g which will be less than h.

$f' \qquad g' \qquad h'$

$\frac{1+x^2-(2x^2)}{(1+x^2)^2} \qquad \frac{1}{1+x^2} \qquad 1$

$\frac{1-x^2}{(1+x^2)^2} \overset{?}{<} \frac{1}{1+x^2} \qquad \frac{1}{1+x^2} \overset{?}{<} 1$

$1-x^2 < 1+x^2 \qquad 1 < 1+x^2$

$0 < 2x^2 \qquad 0 < x^2$

True $\forall x > 0$ True

from point A, $f' < g' < h'$ so $f < g < h$.

IV

a) $\lim_{x\to a} \frac{x^x - a^x}{a^x - x^a} \quad \frac{0}{0}$

$y = x^x \qquad y = a^x$

$\ln y = x \ln x \qquad y' = a^x \ln a$

$\frac{1}{y} y' = \ln x + 1$

$y' = x^x(\ln x + 1)$

$\lim_{x\to a} \frac{x^x(\ln x+1) - a^x \ln a}{a^x \ln a - a x^{a-1}}$

$\frac{a^a(\ln a + 1) - a^a \ln a}{a^a \ln a - a^a}$

$\frac{a^a(\ln a + 1 - \ln a)}{a^a(\ln a - 1)}$

$\frac{1}{\ln a - 1}$

IV

b) $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x} \quad \frac{0}{0}$

$\lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1} \quad \frac{0}{0}$

$\lim_{x\to 0} \frac{-\sin x}{2\sec^2 x \tan x}$

$\lim_{x\to 0} \frac{-\sin x}{\frac{1}{\cos^2 x} \cdot \sin x}$

$-\frac{1}{2}$

c) $\lim_{x\to 0} x^{\frac{2}{\ln x}}$

$\lim_{x\to 0} x^{2\log_x e}$

$\lim_{x\to 0} x^{\log_x e^2}$

$\lim_{x\to 0} e^2$

e^2

12/7/95

Brooke Burton 94
John Moore 82
Roxanne Clark 81
Brad Horn 79
Steve Soggs 68
Corey Anderson 56
Nathan Hughes 55
Chrystelle Ocampo 50
Rob Thayer 40

Great team.
Needs a bit of
cosmetic repairs.

12/29/98

Nicole Sammel 95
Ami Phillippi 90
Matt Marchand 87
Nicole Gonzales 77
Eric Feldson 65

12/29/2000

Robert Green 99
Robert Delgado 98
Elaine Mau 95
Joe Mantell 93
Tracy Bunting 89
Esmeralda Gonzales 83
Lisa Irwin 77
Steven Schleich Harris 77
Brian Ortiz 71
Ryan Ferguson 70
Melanie Bandour 70
Santa Clarita 66
Melissa Santa Maria 54

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept*!*

95/98/00/02

Chapter 6 Mechanical Make Up Exam

I Given $f(x) = 2\sin^{-1} x$, $g(x) = \cos^{-1}\left(1 - 2x^2\right)$ and $0 \le x \le 1$ (30 pts tot)

a) Find the ***exact*** X value which is guaranteed by the Law of the Mean for $f(x)$ for the given interval.

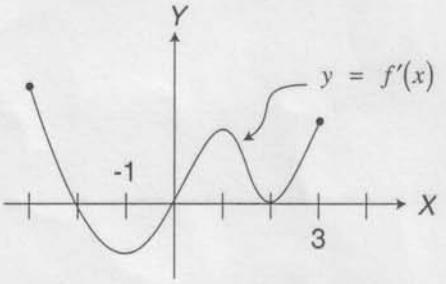
b) Convert your answer to part "a" to a decimal accurate to 3 places.

c) Find $f'(X)$. (Remember, X is a number. I want f' evaluated at X.)

d) Find $g(0)$, $g(1)$ and $g'(x)$.

e) What speculations or conclusions could be made concerning the functions f and g ?

II The figure on the right shows the graph of f', **(the derivative of the function f).**



Note: This is the graph of the **derivative of f**, not the graph of f

The domain of the function f is the closed interval [-3, 3].

a) For what values of x does f have extrema? Identify those x-values. Justify, of course.

b) For what values of x is the graph of f concave up? Justify. (30 pts tot)

c) Use the information found in parts "a" and "b" and the fact that $f(-3) = 0$ to sketch a possible graph of f . Be especially sure to specifically indicate *any* significant points.

III Prove: $\dfrac{x}{1 + x^2} < \tan^{-1} x < x$ for $x > 0$. (20 pts tot)

IV Evaluate the following limits: (20pts tot)

a) $\lim\limits_{x \to a} \dfrac{x^x - a^x}{a^x - x^a}$ b) $\lim\limits_{x \to 0} \dfrac{\sin x - x}{\tan x - x}$ c) $\lim\limits_{x \to 0^+} x^{\frac{2}{\ln x}}$

I

a) $\lim_{x\to 1} \frac{x-1}{\sqrt{x^2+3}-2}$ $\quad \frac{1-1}{2-2}=\frac{0}{0}$

$\lim_{x\to 1} \frac{1}{\frac{x}{\sqrt{x^2+3}}} = \lim_{x\to 1} \frac{\sqrt{x^2+3}}{x}$

$= \frac{2}{1} = 2$

b) $\lim_{x\to e} \frac{e^x - x^e}{x^x - e^e}$ $\quad \frac{e^e-e^e}{e^e-e^e}=\frac{0}{0}$

$\lim_{x\to e} \frac{e^x - e x^{e-1}}{x^x(\ln x+1)}$ $\quad y=x^x$, $\ln y = x\ln x$, $y' = x^x(\ln x+1)$

$\frac{e^e - e e^{e-1}}{e^e(2)} = \frac{e^e - e^e}{2e^e} = 0$

II $f(x) = \frac{x^3-x}{x^2+x} = \frac{x(x^2-1)}{x(x+1)} = \frac{x^2-1}{x+1}$

a) $\lim_{x\to 0} \frac{x^2-1}{x^2-4} = \frac{-1}{-4} = \frac{1}{4}$

b) $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x}$ $\quad \frac{0-0}{0-0}=\frac{0}{0}$

$\lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1}$ $\quad \frac{1-1}{1-1}=\frac{0}{0}$

$\lim_{x\to 0} \frac{-\sin x}{2\sec^2 x \tan x} = \lim_{x\to 0} \frac{-\cos^3 x}{2} = -\frac{1}{2}$

c) $f(x) = \frac{x^2-1}{x^2-4} \neq 0$

$x \neq \{\pm 1\}$

Vertical asymptotes of Non-Reduced form Domain Restrictions

$x=-2,\ x=2$

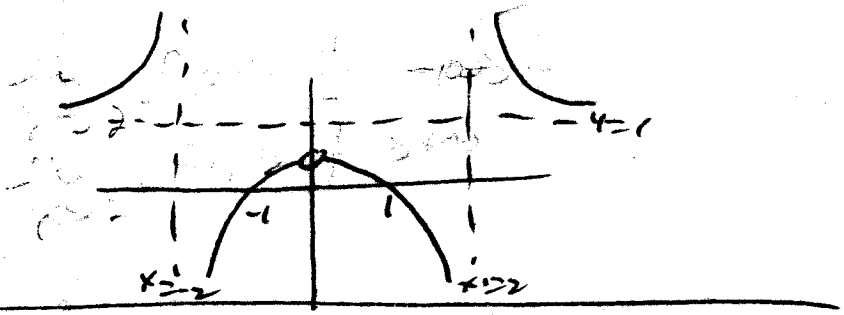
Horizontal $\lim_{x\to\pm\infty} f(x)$

$\lim_{x\to\pm\infty} \frac{x^2-1}{x^2-4} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} =$

$\lim_{x\to\pm\infty} \frac{1-\frac{1}{x^2}}{1-\frac{4}{x^2}} = 1 \therefore y=1$

d) f is even $f(-x)=f(x)$ $\therefore$ reflects across y-axis

e) $\lim_{x\to 2^-} f(x) \to \frac{+}{-} \to -\infty$ $\quad \lim_{x\to 2^+} f(x) = \frac{+}{+} \to +\infty$



III $f(x) = 3x+1-x^2$ $(1,3)$

$f'(x) = 3-2x \Big|_{x=1} = 1 = m$

$y-3 = 1(x-1)$

$y = x+2$ tangent line.

Let $g(x) = 3x+1-x^2-(x+2)$

$= -x^2+2x-1$

WTS $g(x) \leq 0$ ie, $g(x)$ is NOT > 0.

$g(x) = -(x^2-2x+1) = -(x-1)^2$

$\Rightarrow$ Since $g(x)$ is the difference between f and the tangent line, and $g(x)$ is <0 $\forall x \neq 1$, ie $g(x)$ is NOT >0 for any x, then f is NOT above the tangent line at $(1,3)$

IV $f(x) = \frac{(x^2+1)}{e^x}$ $-4 \le x \le 4$

a) $f'(x) = \frac{2xe^x - e^x(x^2+1)}{e^{2x}}$

$\frac{2x - x^2 - 1}{e^x}$

$f'(x) \quad -\frac{(x-1)^2}{e^x} = 0$

$- \quad 0 \quad -$

$\nearrow \quad 1 \quad \searrow$

f decreases in interval is

$f(-4) = 17e^4$

$(-4, 17e^4)$ is max

$f(4) = \frac{17}{e^4}$

$(4, \frac{17}{e^4})$ min

$\leftarrow$ NAT dec to it.

b) $f'(x) = \frac{-(x-1)^2}{e^x}$

$f''(x) = \frac{-2(x-1)e^x + (x-1)^2 e^x}{e^{2x}}$

$= \frac{-2x+2+x^2-2x+1}{e^x}$

$= \frac{x^2-4x+3}{e^x} = \frac{(x-1)(x-3)}{e^x}$

$f'' \quad + \quad 0 \quad - \quad 0 \quad +$

$\nearrow \quad 1 \quad \searrow \quad 3 \quad \nearrow$

PI at $(1, \frac{2}{e})$ and $(3, \frac{10}{e^3})$

Extra.

$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$\lim_{x \to 0} \frac{x^2 \sin\frac{1}{x} - 0}{x}$

$\lim_{x \to 0} x \sin\frac{1}{x} = \lim_{x \to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}} < \lim_{x \to 0} \frac{1}{\frac{1}{x}} = 0$

$\lim_{x \to 0} \frac{1}{x} \to +\infty$ $\therefore$ bottom outruns the top. and

and limit exists (and is zero)

Since the limit is the def of the der, the der exists.

11/22/02

	4	5	T
100	0	1	
90	2	2	
80	3	2	
70	3	2	
60	3	3	
59	5	8	

100

25

1121 / 16 = 70.1

1158 / 18 = 64.3

2279 / 34 = 67.0

Many are going to have. to use whim make up here! Too bad for them.

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!
No Calculators, of course.

Name Calc Chp Mech
Per _____ Date 2002

Chapter 6 Mechanical Exam

I Evaluate the following limits. Use l'Hopital's rule at least once, if possible. (tot 20 pts)

A) $\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2}$ B) $\lim_{x \to e} \frac{e^x - x^e}{x^x - e^e}$ C) $\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$

II Given $f(x) = \frac{x^3 - x}{x^3 - 4x}$ (tot 30 pts)

A) Find $\lim_{x \to 0} f(x)$.
B) Find the zeros of f.
C) Write an equation for each vertical and each horizontal asymptote to the graph of f.
D) Describe the symmetry of the graph of f.
E) Sketch the graph of f. Be very sure you explain behavior of graph (use limits).

III Given $f(x) = 3x + 1 - x^2$. Prove that the graph of f is ***not above*** the graph of the tangent line to f at (1,3) for all x. (25 pts)

IV Let f be a function defined by $f(x) = \left(x^2 + 1\right)e^{-x}$ for $-4 \le x \le 4$. (tot 25 pts)

A) What are the coordinates of all extrema? Identify and justify, of course.
B) What are the coordinates of all points of inflection of f. Justify, as usual.

Extra Credit **5 pts**

Let f be the function defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

Using the definition of the derivative, prove that f is differentiable at $x = 0$.

I $y = \sqrt{2\sqrt{x} - \sqrt[3]{x^2}}\Big|_{x=1} = \sqrt{2-1} = 1$

Point $(1,1)$

$m = y' = \frac{1}{2\sqrt{2\sqrt{x} - \sqrt[3]{x^2}}}\left[\frac{2}{2\sqrt{x}} - \frac{2}{3\sqrt[3]{x}}\right]\Big|_{x=1}$

$= \frac{1}{2\sqrt{2-1}}\left[\frac{1}{1} - \frac{2}{3}\right]$

$= \frac{1}{2}\left[\frac{1}{3}\right] = \frac{1}{6}$

$y - 1 = \frac{1}{6}(x-1)$

$y = \frac{1}{6}x + \frac{5}{6}$

II 4) $\lim_{x\to 1} \frac{x-1}{\sqrt{x^2+3}-2}$ $\frac{0}{0}$

$\lim_{x\to 1} \frac{1}{\frac{x}{\sqrt{x^2+3}}} = \lim_{x\to 1} \frac{\sqrt{x^2+3}}{x} = 2$

B) $\lim_{x\to a} \frac{x^x - a^a}{a^x - x^a}$ $\frac{a^a - a^a}{a^a - a^a}$ $\frac{0}{0}$

$x^x = e^{x \ln x}$

$(x^x)' = x^x(\ln x + 1)$, $\lim_{x\to a} \frac{x^x(\ln x + 1) - 0}{a^x(\ln a) - a x^{a-1}}$

$(a^x)' = (e^{\ln a^x})' = a^x \ln a$

$\frac{a^a(\ln a + 1)}{a^a \ln a - a^a}$

$\frac{a^a(\ln a + 1)}{a^a(\ln a - 1)}$

$\frac{\ln a + 1}{\ln a - 1}$

III $f(x) = \frac{x^3 - x}{x^3 - 4x} = \frac{x(x^2-1)}{x(x^2-4)}$

a) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{x^2-1}{x^2-4} = \frac{1}{4}$

B) $f(x) = \frac{x(x^2-1)}{x(x^2-4)} \overset{\text{Set}}{=} 0$

$x \neq 0 \therefore x \in \{\pm 1\}$

c) Vert asymp occur at non-reducible Domain Restrictions.

$x = 2;\ x = -2$

Horizontal asymptote occurs as limit $\to \pm\infty$ or more clearly as denom is divided into numerator

$x^2-4 \overline{)\, x^2-1}$ (1; remainder 3)

$\therefore f(x) = 1 + \frac{3}{x^2-4}$

So $y = 1$ is the horizontal asymp.

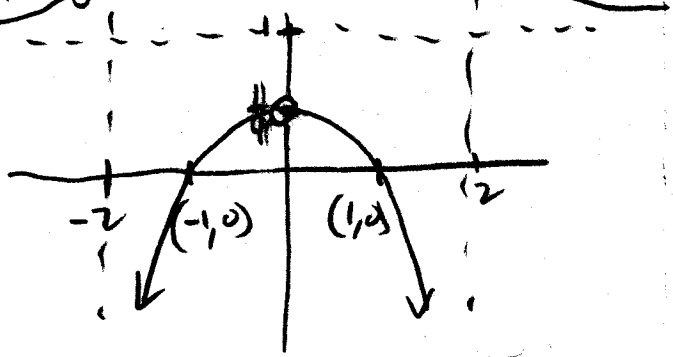
Since $\frac{3}{x^2-4} \to 0$ as $x \to \pm\infty$,

$f(x) \to 1^+$ from TOP

D) f is an Even fctn.

$f(-x) = f(x)$ so f reflects Thru y axis.

E)



IV $f(x) = (x^2+1)e^{-x}$ $-4 \le x < 4$

A) $f'(x) = 2xe^{-x} - (x^2+1)e^{-x}$

$= e^{-x}(2x - x^2 - 1)$

$= -e^{-x}(x^2 - 2x + 1)$

$= -e^{-x}(x-1)^2$

$- \quad + \quad +$

$f' < 0 \ \forall x$

f decreases throughout interval

B) $f' = -e^{-x}(x-1)^2$ (u, v)

$f'' = e^{-x}(x-1)^2 - e^{-x}(2(x-1))$

$= e^{-x}(x-1)[x-1-2]$

$= e^{-x}(x-1)(x-3)$

f'': $+ \ 0 \ - \ 0 \ +$ on $0, 1, 3$

Since concavity does change in each interval, PIs occur at $x=1$, $x=3$.

Extra. $f(-x) = f(x)$ $f(p) = 1$ $f'(p) = 5$

f is even fctn.

$f(x) = ax^{2N}$ for example. There is NO constant term!

$f'(x) = 2anx^{2N-1}$ f' is odd!

ie $g(-a) = -g(a)$ so

Since $f'(p) = 5$

Then $f'(-p) = -f'(p)$

So $f'(-p) = -5$

$f(-4) = 17e^4$

$f(4) = \frac{17}{e^4}$

$\therefore$ Abs max occurs at $x = -4$ and is $17e^4$ because f decreases from it.

V $f(x) = 3x + 1 - x^2$ (1, 3)

Need tangent line.

$f'(x) = 3 - 2x \big|_{x=1} = 1 = m$

$y - 3 = 1(x-1)$

$y = x + 2$ tan.

let $g(x) = 3x + 1 - x^2 - x - 2$

WTS $g(x) \le 0$

$g(x) = -(x^2 - 2x + 1)$

$g(x) = -(x-1)^2$

Since g is diff between f and tangent line, and $g < 0 \ \forall x$, then f is below its tangent line.

b) $f'(0) = 0$ since f is Even. The lowest exponent in f is x^2 f' would have at least $2x^1$, ie f' still has an x in every term. So $f'(0) = 0$,

1991

Remove I it really asks a similar question as Part V. Increase Part II to 30 pts and part III to 30 pts.

1001	0
90	4
80	7
70	
60	3
59	1

79.8

A.P. Calculus

Name Calc C6 mech 1991

Copy original problem.

Convince *me* that **you** understand the concept!

Per _____ Date Becomes

No calculators

Chapter 6 Mechanical Exam

2002

I Find the equation of the tangent line to $y = \sqrt{2\sqrt{x} - \sqrt[3]{x^2}}$ at $x = 1$. (10 pts)

II Evaluate the following limits. Use l'Hopital's rule at least once, if possible. (tot 15 pts)

A) $\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2}$ B) $\lim_{x \to a} \frac{x^x - a^a}{a^x - x^a}$

III Given $F(x) = \frac{x^3 - x}{x^3 - 4x}$ 30 (tot ~~25~~ pts)

A) Find $\lim_{x \to 0} F(x)$.

B) Find the zeros of F.

C) Write an equation for each vertical and each horizontal asymptote to the graph of F.

D) Describe the symmetry of the graph of F.

E) Sketch the graph of F.

IV Let F be a function defined by $F(x) = (x^2+1)e^{-x}$ for $-4 \le x \le 4$. (tot 25 pts)

A) For what value of x does F reach its absolute maximum? Justify, of course.

B) Find the x-coordinates of all points of inflection of F. Justify, as usual.

V Given $F(x) = 3x + 1 - x^2$. Prove that the graph of F is ***not above*** the graph of the tangent line to F at (1,3) for all x. 30 ~~25~~ pts)

Extra Credit 5 pts

For all real numbers x, F is a differentiable function such that $F(-x) = F(x)$. Let $F(p) = 1$ and $F'(p) = 5$ for some $p > 0$.

A) Find $F'(-p)$. Justify!

B) Find $F'(0)$. Justify!

A.P. Calculus Chapter 6 Mechanical 2002

			EXAM CHAPTER 5 # 1 R/C Pg 166 - 171; comment and/or explain EACH of the 7 examples. **Be *very* thorough!** 10/31	# 2 Pg 171 - 172 1, 3, 5, 8 - 16, 19 #10 "D" ans: $\text{Sec}^{-1}\, 2x$ # 19 is important Explain # 5 carefully 11 / 1
# 3 Pg 172 20 Pg 177 1, 3, 6 - 9, 11 - 14, 16, 17 Pblms 16 and 17 are very important! 11 / 4	# 4 Pg 177 - 178 18 - 21 Hint 18: see pg 83 Hint 20A multiply by (1+x) Pblm 20 C use calculator to get all 3 digits 11 / 5	# 5 Pg 200 1, 4, 5, 8, 9, 25 - 32 For pblms 25 - 32; ALSO state coordinates for *ALL* extrema. 11 / 6	# 6 Pg 178 22, 23 # 7 Pg 181- 182 1 - 14, 16 - 22 11/ 7	R/C Pg 232 - 239 # 8 • • • • • • • • • • • • • • • Pg 239 1 - 20 # 9 11/ 8
No School today # 10 Pg 215 2, 3, 4, 7 - 9, 15, 18 for pblm 18 only, make f, f', f'' graphs	# 11 Pg 239-240 21 - 24, 26 - 35, 39 ***ALSO*** Find: the two limits listed at the bottom of the sked	# 12, 13 GQ Ch6 Pg 241 - 242 1 - 3, 5, 7 GQ Ch6 Pg 241 - 242 10 - 13, 19AB GQ Ch 1 - 6 Comp Pg 243 1, 2, 3 11/ 13	# 14 GQ Ch 1-6 Pg 243 4 - 9, 11 - 15 GQ Ch 1-6 Concepts Pg 244 3, 4, 5 # 15 RE Ch 1 - 6 Pg 244 - 249 1 - 26, 33, 34, 44 11/ 14	**# 16, 17, 18, 19,20** **Pick up** **Continuous Enjoyment:** ***Four Corners of the World* edition** 11/ 15
11/18	11/ 19	11/ 20	Put last year's exam on the chalk boards. 11/ 21	**Chapter 6 Mechanical Exam!!!** 11/22
	$\lim_{x \uparrow \pi} (1 - \sin x)^{\cot x}$	$\lim_{x \uparrow \frac{\pi}{2}} (1 + \cos x)^{\tan x}$	**Find those analysis notes !!!!!!** **I *just remembered* a few things we didn't cover last year.** **You're going to *just* love this!**	

I 5 pts ea

A) $\lim_{x\to 1} \frac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$ $\frac{1-1}{1-1} \frac{0}{0}$ l'hopital

$\lim_{x\to 1} \frac{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{4}x^{-\frac{3}{4}}}{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{5}x^{-\frac{4}{5}}}$

$\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} - \frac{1}{5}}$ $\frac{4-3}{12}$ $\frac{1}{12}$

$\frac{5-3}{15}$ $\frac{2}{15}$

$\frac{1}{12} \cdot \frac{15}{2} = \frac{5}{8}$

B) $\lim_{x \uparrow \frac{\pi}{2}} (1-\sin x)^{\cot x}$

$\lim_{x\to\frac{\pi}{2}^-} \ln Y = \cot x \ln(1-\sin x)$

$\lim_{x\to\frac{\pi}{2}^-} \frac{\ln(1-\sin x)}{\tan x}$ $\frac{-\infty}{\infty}$ l'hop.

$\lim_{x\to\frac{\pi}{2}^-} \frac{\frac{-\cos x}{1-\sin x}}{\sec^2 x} = \lim_{x\to\frac{\pi}{2}^-} \frac{-\cos^3 x}{1-\sin x}$ $\frac{0}{0}$

$\lim_{x\to\frac{\pi}{2}^-} \frac{-3\cos^2 x(-\sin x)}{-\cos x} = \lim_{x\to\frac{\pi}{2}^-} -3\cos x \sin x = 0$

$\therefore \ln Y = 0 \quad \therefore Y = e^0 \quad \therefore Y = 1$

C) $\lim_{x\to 0} \frac{x - \sin x}{x + \sin x}$ $\frac{0-0}{0+0}$ $\frac{0}{0}$ l'hopital.

$\lim_{x\to 0} \frac{1-\cos x}{1+\cos x}$ $\frac{1-1}{1+1} = \frac{0}{2} = 0$

D) $\lim_{x\to\infty} \frac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{2}}}$ $\frac{\frac{1}{3}}{\frac{1}{3}}$ l'hopital as like A" But won't improve ∴ do without l'hopital.

$\lim_{x\to\infty} \frac{1 - \frac{1}{x^{\frac{1}{12}}}}{1 - \frac{1}{x^{\frac{3}{15}}}} = \frac{1}{1} = 1$

II total 30 pts

4) WTS $0 < \frac{a}{a+b} < 1$

$0 < a$ given

5 $0 < \frac{a}{a} = 1$

$a < a+b$ $0 < \frac{a}{a+b} < \frac{a}{a} = 1$

$\therefore \frac{a}{a+b} < 1$ $0 < \frac{a}{a+b} < 1$

B) $f(x) = |x| = \sqrt{x^2}$ 5 pts

$f'(x) = \frac{1}{2}(x^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{|x|}$

C) $G(x) = |x|^a |x-1|^b$

$G'(x) = |x-1|^b a|x|^{a-1} d(|x|) + |x|^a b|x-1|^{b-1} d(|x-1|)$

$= a|x|^{a-1}|x-1|^b \frac{x}{|x|} + |x|^a b|x-1|^{b-1}\frac{(x-1)}{|x-1|}$

$= ax|x|^{a-2}|x-1|^b + b|x|^a|x-1|^{b-2}(x-1)$

$= |x|^{a-2}|x-1|^{b-2}[ax|x-1|^2 + b|x|^2(x-1)]$

$= |x|^{a-2}|x-1|^{b-2}[(x-1)x(a(x-1)+bx)]$

$= |x|^{a-2}|x-1|^{b-2}[(x-1)x(ax-a+bx)]$

$\times(a+b)$

D) f' $\overset{0}{-}$ $\overset{\frac{a}{a+b}}{+}$ $\overset{1}{0}$ $\overset{1}{-}$ $\overset{1}{0}$ $\overset{}{+}$ Rel max at $x = \frac{a}{a+b}$

10 pts f' cont

neg dropped due to abs.

E) $f(\frac{a}{a+b}) = \left|\frac{a}{a+b}\right|^a \cdot \left|\frac{a}{a+b} - 1\right|^b$

$\frac{a^a}{(a+b)^a}\left|\frac{a-a-b}{(a+b)}\right|^b$

$\frac{a^a}{(a+b)^a}\left(\frac{b^b}{(a+b)^b}\right)$

$\frac{a^a b^b}{(a+b)^{a+b}}$ 5 pts.

III $f(x) = \frac{x}{(x-1)^2}$ u v

$\lim_{x\to\pm\infty} f(x) = \frac{\infty}{\infty}$ $\lim_{x\to\pm\infty} \frac{1}{2(x-1)} \Rightarrow 0$, $y=0$; $x=1$

$f'(x) = \frac{(x-1)^2 - x\cdot 2(x-1)}{(x-1)^4}$

$= \frac{(x-1)-2x}{(x-1)^3}$

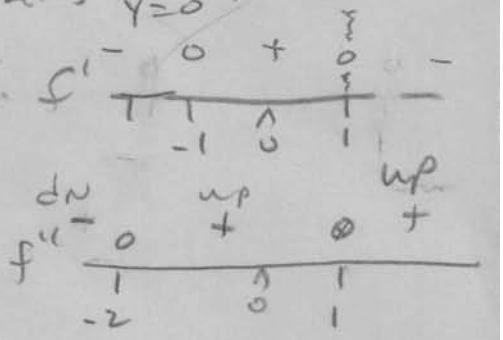
$= -\frac{x+1}{(x-1)^3}$ u v

$f''(x) = -\left[\frac{(x-1)^3 - (x+1)3(x-1)^2}{(x-1)^6}\right]$

$= -\left[\frac{x-1-3-3x}{(x-1)^4}\right]$

$= -\left[\frac{-2x-4}{(x-1)^4}\right]$

$= \frac{2(x+2)}{(x-1)^4}$



min at $x=-1$ f' test.

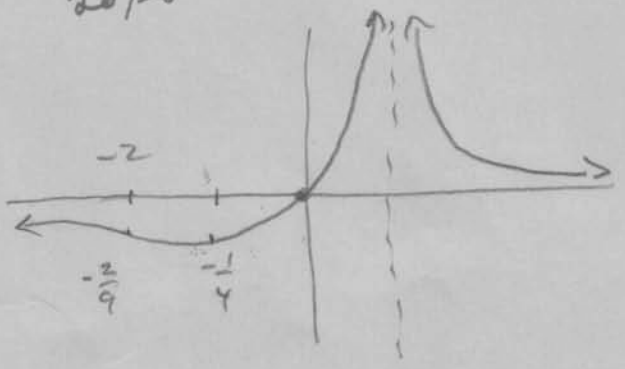
$f(-1) = \frac{-1}{(-2)^2} = \frac{-1}{4}$

$\lim_{x\to 0^-} f(x) = \frac{1}{-0} \to -\infty$

$\lim_{x\to 0^+} f(x) = \frac{1}{+0} \to +\infty$

$f(-2) = \frac{-2}{9}$

20 pts



IV $f(x) = x^x = e^{x\ln x}$ u v

$f'(x) = x^x\left[\ln x + 1\right] \overset{set}{=} 0$

$\ln x = -1$

$x = \frac{1}{e}$

	-	0	+
	∄		
	?	$\frac{1}{e}$	

min at $\frac{1}{e}$ f' test. 10 pts

V $\lim_{x\to 1} x^{\frac{1}{\ln x}}$ $\ln 1 = 0$, $\frac{1}{0} = \infty$

so it heads like 1. value 3 pts

Notice $\ln x \equiv \frac{1}{\log_x e}$ So

$\lim_{x\to 1} x^{\log_x e} = \lim_{x\to 1} e = e$ 10 pts.

VI $f(x) = \frac{\sin x}{x}$ u v WTP $f'(x) < 0$

$f'(x) = \frac{x\cos x - \sin x}{x^2} \overset{set}{=} 0$

$x^2 > 0$ 10 pts.

WTP $x\cos x - \sin x < 0$

$g(x) = x\cos x - \sin x$ u v

$g'(x) = \cos x - x\sin x - \cos x$

$= -x\sin x$

$- + + = - < 0$

$g(0) = 0 - 0 = 0$ $\therefore$ g decreases from zero $\therefore$ $g < 0$ $\therefore$ $f' = \frac{-}{+} < 0$

4 pts $\therefore$ $f' < 0$ $\therefore$ f decreases.

Extra credit

	A	B	C	D
A	-		21	9
B		-	5	7
C	21	5	-	12
D	9	7	12	-

order:
A D B C
dist 16 miles.

AD + DC = AC
D between A, C
BD + BC = CD
B between C, D

A D B C
9 7
16 5
21

1987 Tough exam
I used it for their make up as well.
Then I'll see if they learned anything.

regions same year as make up

86
75
67
60
58 -2
57
55
54
46
42
37

57, 92 Are

96 -2
95
93
91 -2 87.4
85
82
84
71

Eliminate I B, C, make part I 10 pts ea.

Part II

Keep A, B.

change part c to Given $G'(x) =$ . . . .

to include part D.

then renumber part E to D.

make EXTRA credit.

derive ~~Prove that~~ the Derivative given in Part II c ~~is car~~

ie actually 'take Der of G.

CALCULUS Chapter 6 Mechanical Name C 6mech/C87

11/12/87 89

Be sure YOUR method is clear to ME.

I Find the following limits. Use L'Hopital's rule if possible. (Hint: All exist) (5 pts ea)

A) $\lim_{X \to 1} \dfrac{X^{\frac{1}{3}} - X^{\frac{1}{4}}}{X^{\frac{1}{3}} - X^{\frac{1}{5}}}$ B) $\lim_{X \uparrow \frac{\pi}{2}} (1 - \sin X)^{\cot X}$ C) $\lim_{X \to 0} \dfrac{X - \sin X}{X + \sin X}$

D) $\lim_{X \to \infty} \dfrac{X^{\frac{1}{3}} - X^{\frac{1}{4}}}{X^{\frac{1}{3}} - X^{\frac{1}{5}}}$

II Given the real numbers $a, b > 0$ (30 pts)

A) Argue that $0 < \dfrac{a}{a+b} < 1$

B) Given $F(X) = |X|$, Using the fact that $F(X) = \sqrt{X^2}$, Show that $F'(X) = \dfrac{X}{|X|}$

C) Given $G(X) = |X|^a |X-1|^b$

Show that $G'(X) = |X|^{a-2} |X-1|^{b-2} X (X-1)(a+b)\left[X - \dfrac{a}{a+b}\right]$

D) For which X does G have a relative maximum?

E) Show that this relative maximum is: $\dfrac{a^a b^b}{(a+b)^{a+b}}$

III Given $F(X) = \dfrac{X}{(X-1)^2}$, Determine extrema and draw the graph of F. (20 pts)

IV For which $X > 0$ does $F(X) = X^X$ achieve its minimum value? (10 pts)

V Determine: $\lim_{X \to 1} X^{\frac{1}{\ln X}}$ Explain your reasoning. (10 pts)

VI Prove that $F(X) = \dfrac{\sin X}{X}$ is a decreasing function. for $0 < x < \frac{\pi}{2}$ (10 pts)

EXTRA CREDIT ----------5 pts---

	A	B	C	D
A			21	
B				7
C		5		12
D	9			

Towns: A, B, C, D lie on a single railroad line (in some order). The diagram is a partially filled in mileage chart. How far is it from Town A to Town B and what is the order of the towns?

I

1) $\lim_{x\to 1} \frac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$ $\frac{0}{0}$

$\lim_{x\to 1} \frac{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{4}x^{-\frac{3}{4}}}{\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{5}x^{-\frac{4}{5}}}$

$\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} - \frac{1}{5}} = \frac{\frac{4-3}{12}}{\frac{5-3}{15}}$

$\frac{1}{12} \cdot \frac{15}{2} = \frac{5}{8}$

12) $\lim_{x\to\infty} \frac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}} \cdot \frac{\frac{1}{x^{\frac{1}{3}}}}{\frac{1}{x^{\frac{1}{3}}}}$

$\lim_{x\to\infty} \frac{1 - x^{-\frac{1}{12}}}{1 - x^{-\frac{2}{15}}} = 1$

$x^{\frac{1}{4} - \frac{1}{3}}$ $\frac{3-4}{12}$

$x^{\frac{1}{5} - \frac{1}{3}}$ $\frac{3-5}{15}$

c) $\lim_{x\to 1} x^{\frac{1}{\ln x}}$

$\lim_{x\to 1} x^{\log_x e} = e$

II

A) $a>0 \quad b>0$

$0 < \frac{a}{a} = 1$

$0 < \frac{a}{a+b} < \frac{a}{a}$ since $a+b > a$

$0 < \frac{a}{a+b} < 1$

13) $f(x) = |x| = \sqrt{x^2}$

$f(x) = (x^2)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2)^{-\frac{1}{2}} \cdot 2x$

$= \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$

c) $G(x) = |x|^a |x-1|^b$

$G'(x) = |x|^{a-2} |x-1|^{b-2} x(x-1)(a+b)\left[x - \frac{a}{a+b}\right] \overset{\text{set}}{=} 0$

max at $\frac{a}{a+b}$ f' ~~cut~~.

G' : $-$ 0 $+$ 0 $-$ 0 $+$ over 0, $\frac{a}{a+b}$, 1

$G\left(\frac{a}{a+b}\right) = \left|\frac{a}{a+b}\right|^a \left|\frac{a}{a+b} - 1\right|^b$

$\frac{a^a}{(a+b)^a} \cdot \left|\left(\frac{a-a-b}{a+b}\right)^b\right|$

$\frac{a^a}{(a+b)^a} \left|\left(\frac{-b}{(a+b)^b}\right)\right|$

$\rightarrow \frac{a^a b^b}{(a+b)^{a+b}}$

Note NEG sign dropped because of absolute value sign

III $f(x) = \frac{x}{(x-1)^2}$

$f'(x) = \frac{(x-1)^2 - x\,2(x-1)}{(x-1)^4}$

$= \frac{x^2 - 2x + 1 - 2x^2 + 2x}{(x-1)^4}$

$= \frac{1 - x^2}{(x-1)^4}$

$= \frac{-(x-1)(x+1)}{(x-1)^4}$

$f'(x) = -\frac{(x+1)}{(x-1)^3}$

min at $x = -1$

$f(-1) = \frac{-1}{4}$

$f(-2) = \frac{-2}{9}$

$\lim_{x\to\pm\infty} f(x) = \frac{\infty}{\infty}$

$\lim_{x\to\pm\infty} \frac{1}{2(x-1)} \Rightarrow 0$

f': $-$ 0 $+$ ∄ $-$ (at -1, 0, 1)

$f''(x) = -\left[\frac{(x-1)^3 - (x+1)3(x-1)^2}{(x-1)^6}\right]$

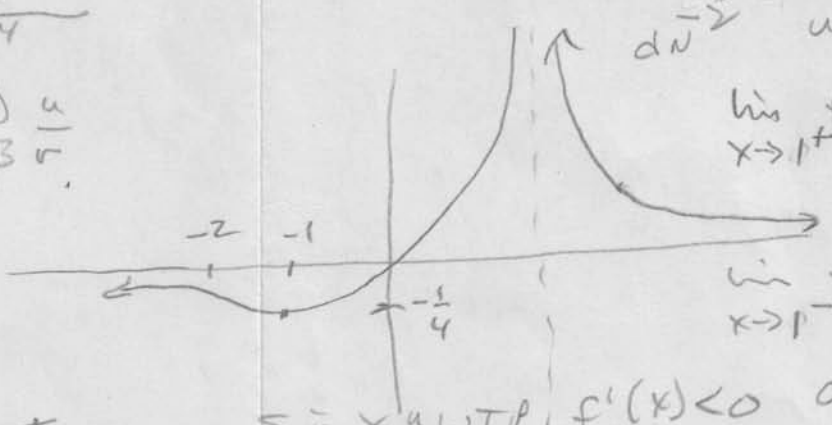
$= -\left[\frac{x-1-(3x+3)}{(x-1)^4}\right] = -\left[\frac{-2x-4}{(x-1)^4}\right]$

$f'' = \frac{2x+4}{(x-1)^4}$ f'': $-$ 0 $+$ ∄ $+$ (at -2, -1, 1)

dn dn up up

$\lim_{x\to 1^+} f(x) \Rightarrow +\infty$

$\lim_{x\to 1^-} f(x) \Rightarrow +\infty$



IV $f(x) = x^x$

let $y = x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$

$\frac{dy}{dx} = x^x(\ln x + 1) \overset{\text{set}}{=} 0$

$\ln x = -1$

$x = \frac{1}{e}$

f': $-$ 0 $+$ (at 0, $\frac{1}{e}$)

min at $x = \frac{1}{e}$ f' test

V $f(x) = \frac{\sin x}{x}$ WTP: $f'(x) < 0$ $0 < x < \frac{\pi}{2}$

$f'(x) = \frac{x\cos x - \sin x}{x^2} \overset{\text{set}}{=} 0$ wrt top < 0 since $x^2 > 0$

$x\cos x - \sin x < 0$

let $g(x) = x\cos x - \sin x$

$g(0) = 0$

$g'(x) = \cos x - x\sin x - \cos x$

$g'(x) = -x\sin x$

$-\ +\ +\ = -\ < 0$

so $g'(x) < 0$

$\Rightarrow$ g decreases fm 0

$\therefore g < 0 \therefore f' < 0 \therefore$

f is a decreasing fctn

Extra credit

$G(x) = |x|^a |x-1|^b$

$G'(x) = |x-1|^b a|x|^{a-1}\frac{d(|x|)}{dx} + |x|^a b|x-1|^{b-1}\frac{d(|x-1|)}{dx}$

$= |x-1|^b a|x|^{a-1}\frac{x}{|x|} + |x|^a b|x-1|^{b-1}\frac{(x-1)}{|x-1|}$

$= ax|x-1|^b|x|^{a-2} + b(x-1)|x|^a|x-1|^{b-2}$

$= |x|^{a-2}|x-1|^{b-2}\left[ax(x-1)^2 + b(x-1)x^2\right]$

$= |x|^{a-2}|x-1|^{b-2}(x-1)\left[ax(x-1) + bx^2\right]$

$= |x|^{a-2}|x-1|^{b-2}(x-1)\left[ax^2 - ax + bx^2\right]$

$= |x|^{a-2}|x-1|^{b-2}x(x-1)\left[ax - a + bx\right]$

$= |x|^{a-2}|x-1|^{b-2}x(x-1)\left[x(a+b) - a\right]$

$= |x|^{a-2}|x-1|^{b-2}x(x-1)(a+b)\left[x - \frac{a}{a+b}\right]$

89

100	0	
90	4	94
80	2	
70	7	
60	0	
59	1	57

81

Changes as shown on other side.

Good exam.

ADVANCED PLACEMENT CALCULUS AB
Convince ME that YOU know the concept !
Copy original problem. SHOW ALL WORK !

NAME Calc Ch6 mech 1989

PER _____ DATE _____

CHAPTER 6 MECHANICAL EXAM

I Find the following limits. (All limits DO exist.) Be sure your reasoning is clear. (10 pts ea)

A) $\lim_{x\to 1} \dfrac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$

B) $\lim_{x\to \infty} \dfrac{x^{\frac{1}{3}} - x^{\frac{1}{4}}}{x^{\frac{1}{3}} - x^{\frac{1}{5}}}$

C) $\lim_{x\to 1} x^{\frac{1}{\ln x}}$

Reduce exponents 1 point in size. Like

II Given the real numbers a , $b > 0$ (30 pts total)

A) Argue that : $0 < \dfrac{a}{a+b} < 1$

B) Given $F(x) = |x|$. Using the fact that $F(x) = \sqrt{x^2}$, show that $F'(x) = \dfrac{x}{|x|}$

C) Given $G(x) = |x|^a |x-1|^b$ AND

$$G'(x) = |x|^{a-2} |x-1|^{b-2} x(x-1)(a+b)\left[x - \frac{a}{a+b}\right]$$

1) For which x does G have a relative maximum?

2) Show that this relative maximum value is $\dfrac{a^a b^b}{(a+b)^{a+b}}$

III Given $F(x) = \dfrac{x}{(x-1)^2}$ Determine extrema and draw the graph of F. (20 pts)

IV For which $x > 0$ does $F(x) = x^x$ achieve its minimum value ? (10 pts)

V Prove that $F(x) = \dfrac{\sin x}{x}$ is a DECREASING function for $0 < x < \dfrac{\pi}{2}$ (10 pts)

EXTRA CREDIT -------- 5 points

Show that the derivative of $G(x)$ in part II C is as it is stated.

I A) $\lim_{x\to\infty} \frac{e^x \sin e^{-x}}{x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{e^x \sin e^{-x} + e^x(-e^{-x})\cos e^{-x}}{1}$

$\infty \cdot 0 - \cos 0 = -1$ 5

C) $\lim_{x \uparrow \frac{\pi}{2}} (1+\cos x)^{\tan x}$

$\lim_{x \uparrow \frac{\pi}{2}} \left[(1+\cos x)^{\frac{1}{\cos x}}\right]^{\sin x}$

$e^{\sin \frac{\pi}{2}} = e^1 = e$ 5

D) $\lim_{x \to \frac{\pi}{2}^-} (1-\sin x)^{\cot x}$

$(1-1)^{\frac{0}{1}}$

$0^0 = 1$

$\lim_{x\to\frac{\pi}{2}^-} \left[\left(1+\sin(-x)\right)^{\frac{1}{\sin(-x)}}\right]^{-\cos x}$

e^{-0}

$= 1$

$= 1$ 5

B) $\lim_{x\to\infty} x^{\frac{1}{x}}$ Let $y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x} \ln x$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\therefore \lim_{x\to\infty} \ln y = 0$

But $y = e^0 \therefore y = 1$

So $\lim_{x\to\infty} x^{\frac{1}{x}} = 1$ 5

II A WTP $\frac{x}{1+x^2} < \tan^{-1} x < x \quad \forall x > 0$

$f \quad g \quad h$

$f(A) = g(A) = h(A) = 0, \ A = 0$

If $f' < g' < h'$ then $\forall x > A$

f increases slower than g which increases slower than h so f will be $< g$ which will be $< h$

WTP $f' < g' < h'$

f'	g'	h'
$\frac{1+x^2-(x\cdot 2x)}{(1+x^2)^2}$	$\frac{1}{1+x^2}$	1

$\frac{1-x^2}{(1+x^2)^2} \overset{?}{<} \frac{1}{1+x^2}$ $\quad \frac{1}{1+x^2} \overset{?}{<} 1$

$1-x^2 < 1+x^2$ $\quad 1 < 1+x^2$

$0 < 2x^2$ $\quad 0 < x^2$

True $\forall x$ $\quad$ True $\forall x$

15 pts

From pt A, $f' < g' < h'$ so $f < g < h$. Since the infimum is zero we can conclude $f < g < h \ \forall x > 0$.

II B $x^5+2x^3-2=0$ $[0,1]$ $f(x)=x^5+2x^3-2$

1st we show at least one root
2nd we show less than 2 roots
Conclusion Exactly 1 root.

$f(0)=-2$ $f(1)=1$ $\therefore$ by intermediate Value Thm at least 1 root. 25 pts

assume Two roots $f(p)=f(q)=0$

by Rolle's Thm $\exists$ at least 1 $X \in (a,b)$

$\Rightarrow f'(X)=0$

$f'=5x^4+6x^2$ But $f'>0 \; \forall x \in (0,1)$

So Rolle's Thm fails $\therefore$ $f(p)\neq f(q)$

ie Not 2 roots.

conclusion there is exactly 1 root.

III B $f(x)=x^{\frac{5}{3}}+10x^{\frac{2}{3}}$ f: $-$ (−10) 0 $+$ (0) $+$

20 pts $x^{\frac{2}{3}}(x+10)$

$f'(x)=\frac{5}{3}x^{\frac{2}{3}}+\frac{20}{3}x^{-\frac{1}{3}}$ INC (−4) dec (0) INC

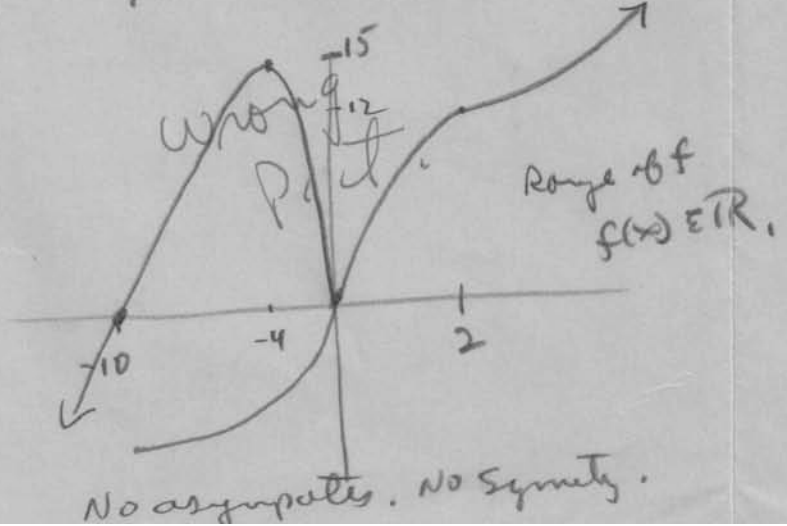
$\frac{5}{3x^{\frac{1}{3}}}(x+4)$

$f''(x)=\frac{10}{9}x^{-\frac{1}{3}}-\frac{20}{9}x^{-\frac{4}{3}}$ dn (0) dn (2) up

$\frac{10}{9x^{\frac{1}{3}}}\left(1-\frac{2}{x}\right)$

$\frac{10}{9x^{\frac{1}{3}}}\left(\frac{x-2}{x}\right)$

rel max $(-4,15)$
rel min $(0,0)$



No asympotes. No Symetry.

III 4) $f(x)=xe^{-x}=\frac{x}{e^x}$ $(0,0)$

$f'(x)=e^{-x}-xe^{-x}=e^{-x}(1-x)$

increase (+) (0) decrease (−)

f' 2 opts. 0 1

$f''(x)=-e^{-x}-(e^{-x}-xe^{-x})$

$=-2e^{-x}+xe^{-x}$

$=e^{-x}(-2+x)$

down (−) (0) up (+)

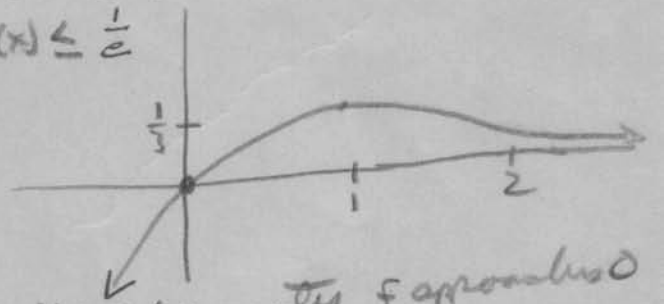
f'' 0 2

$f(1)=\frac{1}{e}\approx\frac{1}{3}$

$\lim_{x\to+\infty} f \Rightarrow 0^+$ $\lim_{x\to-\infty} f(x) \Rightarrow -\infty$

abs max at $(1,\frac{1}{3})$ change in concavity $(2,\frac{2}{e^2})$

R_f $f(x)\leq\frac{1}{e}$



No asympotes f approaches 0 for large x.

No Symetry

1990

Plant 90

Paylor 84

McNally 83

Hartley 81

Land 80

Parker 78

Leary 65

Shannon 58

Foley 47

Snyder 0

Advanced Placement Calculus AB Name Calc Ch 6 mock 1990

Copy the original problem.

Convince *me* that **you** understand the concept. Per _____ Date _____

CHAPTER #6 Mechanical Make - Up

So So Not Up Great

I Limits (5 pts ea)

A) $\lim_{x \to \infty} \frac{e^{x} \operatorname{Sin} e^{-x}}{x}$ B) $\lim_{x \to \infty} x^{\frac{1}{x}}$ C) $\lim_{x \uparrow \frac{\pi}{2}} (1+\operatorname{Cos} x)^{\operatorname{Tan} x}$ D) $\lim_{x \uparrow \frac{\pi}{2}} (1-\operatorname{Sin} x)^{\operatorname{Cot} x}$

II For each of the following, your explanations, justifications, etc. must be clear and easy to follow!

A) $\frac{x}{1+x^2} < \operatorname{Tan}^{-1} x < x \quad \forall\ x > 0$ (15 pts)

B) Show that the equation : $x^5 + 2x^3 - 2 = 0$

has ***exactly*** **one root** in the interval [0 , 1] (25 pts)

III Respond to each of the following for each function listed below (20 pts ea)

Where does F (x) touch or cross the x-axis?
Find the *coordinates* of the critical points.
Show number lines for first and second derivatives.
Identify each critical point as max, min, or PI as well as relative or absolute.
Specifically state the intervals where F is increasing.
Specifically state the intervals where F is concave down.
Sketch the graph of F.
From the graph you draw, state the range of F.

A) $F(x) = xe^{-x}$

(given: $F(1) \approx \frac{1}{3}$)

B) $F(x) = x^{\frac{5}{3}} + 10x^{\frac{2}{3}}$

(given: $F(-4) \approx 15$, $F(2) \approx 12$)

11/21/88

100 ↑	0
90 ↑	5
80 ↑	7
70	8
60	2
59	0

81.23

95

64

Good tough exam.
especially since it was
softened by making
Part IV 2 of 3.
this was a good exam
for all.
maybe I'll make the
make up exam
all 3 sections plus
ext'a credit ...
might be
interesting.

Key 1988

ADVANCED PLACEMENT CALCULUS AB NAME_____

CHAPT 6 MECHANICAL E X A M

I Using l'Hopital's rule, find : (5 pts ea)

A) $\lim_{x \to 0} \frac{xe^x - \text{Six } x}{2x^2}$ B) $\lim_{x \to 0} \frac{3x^2 + \text{Sin } x - x}{x^2}$

II Prove $e^x > 1 + x \quad \forall x < 0$ (15 pts)

III Explain um's. Define each of the following. For each, draw a sketch illustrating your definition. Be sure to indicate on your picture ALL points mentioned. Do NOT use specific numbers. (15 pts total)

A) Rolle's Theorem

B) Law of the Mean (Mean Value Thm)

C) Intermediate Value Theorem.

IV DO ONLY TWO (2) OF THE THREE FOLLOWING PARTS.

For each of the two problems you do, respond to each of the following. (30 pts ea)

Where does F(x) touch or cross the x-axis?
Discuss the symmetry of F(x). (Odd, Even, Neither, and so what?)
Find the coordinates of the critical points.
Show number-lines for F′ and F″.
Identify each critical point as :
absolute max, relative max, absolute min, relative min, or point of inflection.
Specifically state the intervals where F is increasing.
Specifically state the intervals where F is concave down.
Determine the equation(s) of ANY asympotes.
Sketch the graph of F(x).
From the graph of F(x), state the range of F(x).

A) $F(x) = x^4 - 2x^2$ B) $F(x) = x^{\frac{2}{3}} + 1$ C) $F(x) = x + \frac{1}{x}$

EXTRA CREDIT ---------- 5 PTS ----------

Given $F(x) = 3x + 1 - x^2$.

Prove that the graph of F is not above the graph of the tangent line to F at (1,3) for all x.

102

100 − 2 | 3

90 | 5

80 | 5

70 | 7

61 | 1

1763 / 21

83.95

Most lost pts because they were unprepared for the definitions and sig of rolles Thm, Law of Thmean etc.

Practically all need improvement in the area of "Significance of the derivative" stuff. maybe a worksheet would be a good idea.

Advanced Placement Calculus Name Calc Ch 6 mech

Copy original problem. Per _____ 92 Date _____

Convince *me* that **you** understand the concept!

Chapter 6 – Mechanical Exam

I Using l'Hopital's rule, if possible, find: (10 pts ea)

a) $\lim_{x \to 0} \frac{xe^x - \sin x}{2x^2}$

b) $\lim_{x \to 0} \frac{3x^2 + \sin x - x}{x^2}$

II Explain-um's: Define each of the following. For each, draw a sketch illustrating your definition. Be sure to indicate on your picture all points mentioned. Do NOT use specific numbers. (20 pts total)

a) Rolle's Theorem

b) Mean Value Theorem

c) Law of the Mean

d) Intermediate Value Theorem

III DO ONLY TWO (2) OF THE THREE FOLLOWING PROBLEMS.

For *each* of the two problems that you do, respond to each of the following: (30 pts ea)

Where does $f(x)$ touch or cross the x-axis?

Discuss the symmetry of $f(x)$. (odd, even, neither, as well as significance)

Find the coordinates of the critical points.

Show number lines for $f'(x)$ and $f''(x)$.

Identify each critical point as Max, Min, or PI and as relative or absolute.

Specifically state the intervals where $f(x)$ is increasing.

Specifically state the intervals where $f(x)$ is concave down.

Determine the equation(s) of *any* asymptotes.

Sketch the graph of $f(x)$.

From your picture, state the range of $f(x)$.

a) $f(x) = x^4 - 2x^2$ b) $f(x) = x^{\frac{2}{3}} + 1$ c) $f(x) = x + \frac{1}{x}$

Extra Credit ----------5 pts ----------

Given $f(x) = 3x + 1 - x^2$.

Prove that the graph of $f(x)$ is not above the graph of line which is tangent to $f(x)$ at (1,3).

I A) $\lim_{x\to 0} \frac{xe^x - \sin x}{2x^2}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x + xe^x - \cos x}{4x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{2e^x + xe^x + \sin x}{4} = \frac{2+0+0}{4} = \frac{1}{2}$

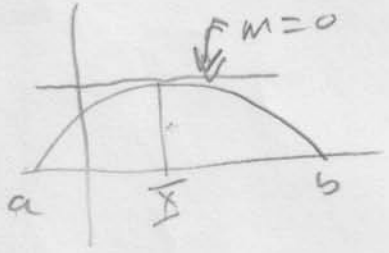
B) $\lim_{x\to 0} \frac{3x^2 + \sin x - x}{x^2}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{6x + \cos x - 1}{2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{6 - \sin x}{2} = \frac{6}{2} = 3$

II A) Rolle's thm.

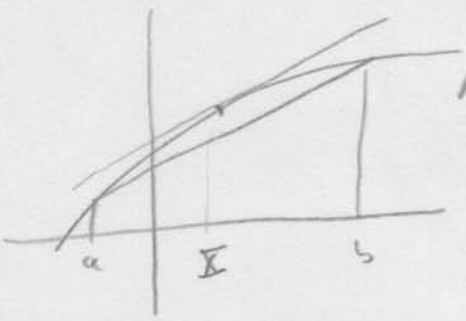
if $f(a) = f(b)$, f cont in $[a,b]$ $\exists\ \bar{x} \in (a,b)$ $\Rightarrow f'(\bar{x}) = 0$



B) mean value thm same as

C) Law of the mean

f is cont in $[a,b]$, f' exists in (a,b) $\exists\ \bar{x} \Rightarrow f'(\bar{x}) = \frac{f(b)-f(a)}{b-a}$



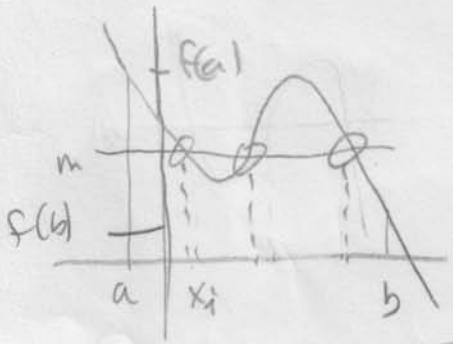
Tangent line // to chord.

D) intermediate Value Thm.

If f in cont in $[a,b]$ and

$f(a) > m > f(b)$ (or vice versa) at least one

then $\exists_{\wedge}\ \bar{x} \Rightarrow f(\bar{x}) = m$



at least 1 x_i guaranteed by IVT.

III A) $f(x) = x^4 - 2x^2 = x^2(x^2-2)$

cross x-axis $0, \pm\sqrt{2}$

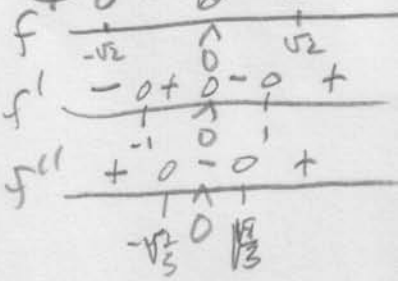
F even fctn sym wrt y axis $\nwarrow$ flects

$f'(x) = 4x^3 - 4x$

$4x(x^2-1)$

$f''(x) = 12x^2 - 4$

$4(3x^2-1)$



$f = x^2(x^2-2)$

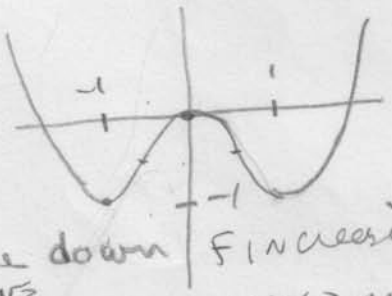
$f(\pm 1) = 1(-1) = -1$

$\lim_{x\to\pm\infty} f(x) \to +\infty$

Domain $x \in \mathbb{R}$,

Range $f(x) \geq -1$

$(-1,0)$ rel max $(-1,-1), (1,-1)$ abs min



concave down $|x| < \frac{\sqrt{3}}{3}$

F increasing $-1 < x < 0$ or $x > 1$

No asymptotes

IV

B) $f(x) = x^{\frac{2}{3}} + 1 \overset{\text{set}}{=} 0$

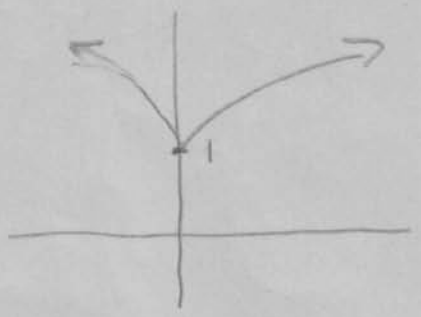
No solution – it DOESN'T cross x-axis

$f(0) = 1$ crosses y axis at $(0, 1)$

$f(-x) = f(x)$ ∴ f is even Symmetric WRT y-axis

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$ Deriv DNE at $x = 0$

$f''(x) = -\frac{2}{9} x^{-\frac{4}{3}}$ x to 4th makes $f'' < 0 \ \forall x$



Range of f: $f(x) \geq 1$

f': $- \ \otimes \ +$ (0) f'': $- \ \otimes \ -$ (0)

No critical pts, f concave down $\forall x \neq 0$

No asympts NO max abs min at $(0, 1)$

f INCREASES $\forall x > 0$ $\lim_{x \to \pm\infty} f(x) = \infty$

C) $f(x) = x + \frac{1}{x}$ f does not touch y-axis $x^2 + 1 = 0$

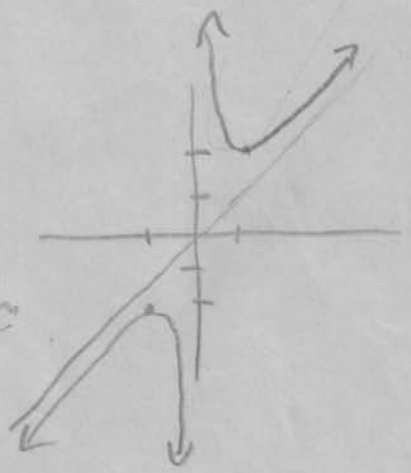
$f(-x) = -f(x)$ f is odd fctn Symmetric WRT origin

$f'(x) = 1 - \frac{1}{x^2} \overset{\text{set}}{=} 0$ $(-1, -2)$

$x^2 - 1 = 0$ $x = \pm 1$ $(1, 2)$

$f''(x) = \frac{1}{x^3}$: $- \ \otimes \ +$ (0)

f': $+ \ 0 \ - \ \otimes \ - \ 0 \ +$ (−1, 0, 1)



Range of f: $|f(x)| \geq 2$

f concave down $\forall x < 0$

f INCREASES $|x| > 1$ Non-reducible Domain restriction

$x = 0$ Vertical asympt

$\lim_{x \to \pm\infty} f(x) \Rightarrow x$ $y = x$ tilted asympt

$\lim_{x \to 0^+} f(x) \Rightarrow +\infty$ $\lim_{x \to 0^-} f(x) = -\infty$

EXTRA $f(x) = 3x + 1 - x^2$ $f' = 3 - 2x \big|_{x=1} = 1$

tangent line: $y - 3 = 1(x - 1)$

$y = x + 2$

Let $g(x) = 3x + 1 - x^2 - x - 2$ WTS $g < 0$

$= -(x^2 - 2x + 1)$

$g(x) = -(x - 1)^2$ Since $g \leq 0 \ \forall x$ and g is diff between f and tangent, f is $\leq$ tangent

86

100	0
90	2
80	1
70	3
60	3
59	15

99

25

5654

Replace pblm I B without which cannot be reduced as easily it will require computer.

AN Excellent exam — over confident Students. they thought last year's analysis would be enough to drag them through — I go to Daily collections of the — Before Discussion they will need to photo copy. Lessons.

It's time for my "You're wasting my time" lecture. the tears will pour tomorrow.

CALCULUS Chapter 6 Mechanical Name Ch 6 mech Calculus 1986

Be sure YOUR method is clear to ME.

I Find the following limits. Use L'Hopital's rule if possible. (Hint: All exist) (5 pts ea)

apply at least once

A) $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$ B) $\lim_{x \uparrow \frac{\pi}{2}} \frac{4 \tan x}{1 + \sec x}$ C) $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$ D) $\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$

II Some people say, "Rolle's Theorem is just a "special case" of the Law of the Mean."

Explain Rolle's Thm, Law of the Mean, and identify the "special case."

What is the "other name" for the Law of the Mean? (15 pts tot)

III Graph the following function. Be sure to show number lines for F′ and F″. Determine the equation of ALL asymptotes. Identify, state coordinates and justify ALL extrema.

$$F(x) = \frac{x^2}{x - 2} \qquad x \in [-2, 6]$$

(25 pts)

IV Given: $F(x) = Ax^3 + Bx^2 + Cx + D$

F has a relative maximum at $(0, \ )$

F changes in concavity at $(1, \frac{1}{3})$ (has PI there)

move back a little - the 1 falls right on the paper crease.

Determine A, B, C, D and Sketch F(x). State coordinates of all extrema, justify, etc. (25 pts)

V Given $X \geq 0$. Prove: $X \geq \ln(1 + X)$ (15 pts)

EXTRA CREDIT ----------5 pts---

Given: $Y = 3X^2 - 12X + 13$

Determine: The equation of the Directorix.

The coordinates of the Focus.

I

A) $\lim_{x\to 0} \frac{e^x+e^{-x}-2}{1-\cos 2x}$ $\frac{1+1-2}{1-1}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x-e^{-x}}{2\sin 2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x+e^{-x}}{4\cos 2x} = \frac{1+1}{4} = \frac{1}{2}$) 5

B) $\lim_{x\uparrow \frac{\pi}{2}} \frac{4\tan x}{1+\sec x}$ $\frac{\infty}{\infty}$

$\lim_{x\uparrow \frac{\pi}{2}} \frac{4\sec^2 x}{\sec x \tan x} = \lim_{x\uparrow \frac{\pi}{2}}$ $\frac{4\sec x}{\tan x} = \frac{\frac{4}{\cos x}}{\frac{\sin x}{\cos x}}$

$\lim_{x\to} \frac{4}{\sin x} = 4$) 5

C) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}$

$\lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$) 5

D) $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x}$ $\frac{0-0}{0-0}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1}$ $\frac{1-1}{1-1} = \frac{0}{0}$

$\lim_{x\to 0} \frac{-\sin x}{2\sec^2 x \tan x}$ $\frac{-\sin y}{2\frac{\sin x}{\cos^3 x}}$

$\lim_{x\to 0} -\frac{1}{2}\cos^3 x - \frac{1}{2}$) 5

II Rolle's 5

Rolle's Thm says if $f(a)=f(b)$ and f is continuous, then $\exists\, X \ni f'(X)=0$

Law of the mean (also called mean value thm) says if f is continuous $\exists\, X \ni f'(X) = \frac{f(b)-f(a)}{b-a}$, ie there is a spot where the slope of the tangent to $f(x)$ = slope of the cord joining the endpts.

The "special case" is when $f(b)=f(a)$, ie a horizontal tangent and chord.

III 25pts

$f(x) = \frac{x^2}{x-2}$ $x \in [-2, 6]$

$x-2 \overline{)x^2}$ $\quad x+2 + \frac{4}{x-2} = f(x)$

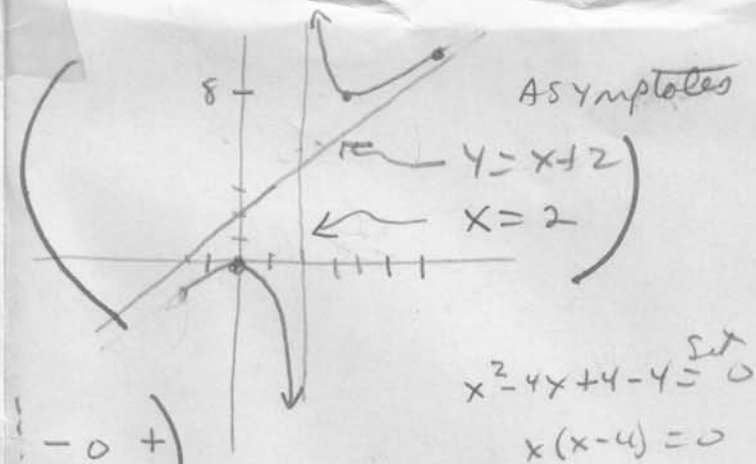
$x^2 - 2x$

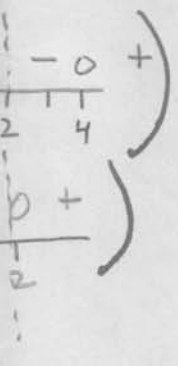
$2x$

$2x - 4$

$f'(x) = 1 - \frac{4}{(x-2)^2}$

$f''(x) = \frac{8}{(x-2)^3}$





$x^2 - 4x + 4 - 4 \overset{\text{set}}{=} 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$

$f(0) = 0$

$f(4) = \frac{16}{2} = 8$

$f(-2) = \frac{4}{-4} = -1$

$f(6) = \frac{36}{4} = 9$

Rel max $(0,0)$ f'' cnct
$(6,9)$ increase to it
Rel min $(-2,-1)$ increase to it
$(4,8)$ f'' cnct

IV 25pts

$f(x) = Ax^3 + Bx^2 + Cx + D$

$f'(x) = 3Ax^2 + 2Bx + C$

$f''(x) = 6Ax + 2B$

$f(0) = 1 \overset{\text{set}}{=} D$

$f'(0) = 0 \overset{\text{set}}{=} C$

$f''(1) = 0 = 6A + 2B$

$f(1) = A + B + C + D = \frac{1}{3}$

$A + B + 0 + 1 = \frac{1}{3}$

$A + B = -\frac{2}{3}$

$3A + B = 0$

$A + B = -\frac{2}{3}$

$2A = \frac{2}{3}$

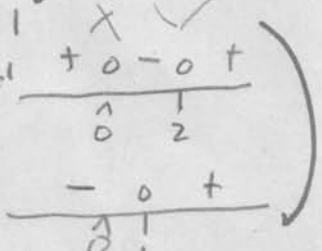
$A = \frac{1}{3} \quad \therefore B = -1$

$f(x) = \frac{1}{3}x^3 - x^2 + 1$

$f'(x) = x^2 - 2x$

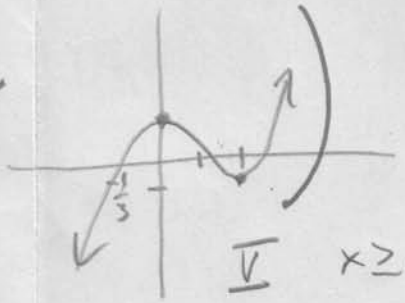
$x(x-2)$

$f''(x) = 2x - 2$



$f(2) = \frac{8}{3} - 4 + 1$ rel MAX $(0,1)$ f'' cnct

$\frac{8}{3} - \frac{9}{3} = -\frac{1}{3}$ rel min $(2, -\frac{1}{3})$ f'' cnct



V $x \geq 0$ WTP $x \geq \ln(1+x)$

Let $f(x) = x - \ln(1+x)$

15pts $f(0) = 0 - 0 = 0$

$f'(x) = 1 - \frac{1}{1+x} = \frac{1+x-1}{1+x} = \frac{x}{1+x}$

$\frac{+}{+} = f'(x) > 0 \ \forall x > 0$, thus

(f is increasing from 0 so
f is > 0, ie $x - \ln(1+x) > 0$
$x > \ln(1+x)$

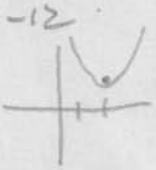
Extra

$y = 3x^2 - 12x + 13$

$3(x^2 - 4x + \ \ ) + 13$

$3(x - 4x + 4) + 13 - 12$

$y = 3(x-2)^2 + 1$



$\frac{1}{4c} = 3 \quad \therefore c = \frac{1}{12}$

focus at $(2, \frac{13}{12})$ Dir $y = \frac{11}{12}$

100	4
90	3
80	7
70	5
60	6
59	4

1944
26(5) 74.8

A lot of weak students?
I think many are capable but
from past yrs so they really
don't get "the system" They just
cover it

Cut I down to 2 10-pt problems

Keep A, B

Advanced Placement Calculus AB

Copy the original problem.

Convince *me* that **you** understand the concept.

Name Calch6 mech 1990

Per _____ Date 1 86

CHAPTER #6 MECHANICAL

I Find the following limits. Apply l'hopital's rule at least once. All exist. ~~(5 pts ea)~~ for each problem 10 pts ea

A) $\lim_{x\to 0} \dfrac{e^{x}+e^{-x}-2}{1+\sec x}$ $1-\cos 2x$

B) $\lim_{x\uparrow \frac{\pi}{2}} \dfrac{4\tan x}{1+\sec x}$

C) $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

D) $\lim_{x\to 0} \dfrac{\sin x - x}{\tan x - x}$

II Some people say, "Rolle's Theorem is just a 'special case' of the Law of the Mean." Briefly ~~Explain~~ Rolle's Thm., Law of the Mean, and Identify the 'special case.' What is the the "other name" for the Law of the Mean? (15 pts tot)

III ~~Graph the following function. Be sure to show number lines for f' and f''.~~ Determine the equation of ~~all~~ asymptotes. Identify, state coordinates and justify *all* extrema. (25 pts)

Any (C) (D)

Given $f(x) = \dfrac{x^2}{x-2}$; $x \in [-2, 6]$

move back

IV Given: $F(x) = Ax^3 + Bx^2 + Cx + D$

F has a relative maximum at $(0, 1)$

F changes concavity at $\left(1, \frac{1}{3}\right)$ (F has a PI there.)

Determine A, B, C, and D. Sketch F (x). State coordinates of all extrema, justify, etc. (25 pts)

V Given $x \geq 0$. Prove: $x \geq \ln(1+x)$ (15 pts)

increase is right on the 1,

EXTRA CREDIT ——— 5 pts

Given: $Y = 3x^2 - 12x + 13$

Determine: The equation of the Directrix.

The coordinates of the Focus.

I a) $\lim_{x\to 0} \frac{e^x+e^{-x}-2}{1-\cos 2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x-e^{-x}}{2\sin 2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x+e^{-x}}{4\cos 2x} = \frac{1+1}{4} = \frac{1}{2}$

b) $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{-\sin x}{2\sec^2 x \tan x}$

$\lim_{x\to 0} \frac{-\sin x}{2\frac{\sin x}{\cos^3 x}}$

$\lim_{x\to 0} \frac{-\cos^3 x}{2} = -\frac{1}{2}$

II Rolle's Thm says if $f(a)=f(b)$ and if f is cont in the interval then $\exists\ \bar{x} \Rightarrow f'(\bar{x})=0$

Law of the mean (AKA mean value thm) says if f is continuous in the interval then $\exists\ \bar{x} \Rightarrow f'(\bar{x}) = \frac{f(b)-f(a)}{b-a}$. it tells us there is at least one spot $\bar{x}$ where the slope of the tangent line is parallel to the slope joining the endpts. The "special case" is when $f(b)=f(a)$ it is a horizontal tangent and slope.

III $f(x) = \frac{x^2}{x-2}$ $x \in [-2, 6]$

$$\begin{array}{r} x+2 \\ x-2 \overline{)\ x^2} \\ x^2-2x \\ \hline 2x \\ 2x-4 \\ \hline 4 \end{array}$$

A) $f(x) = x+2+\frac{4}{x-2}$

B) D_f $x \neq 2$

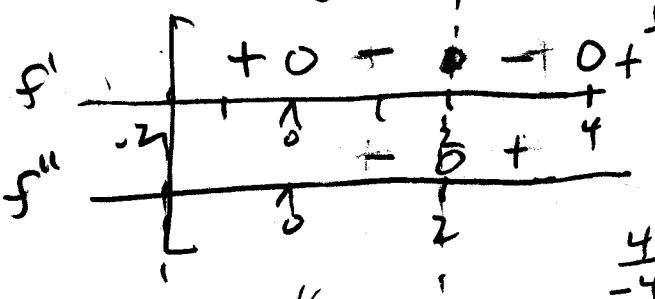
C) $x=2$, $y=x+2$

D) $f'(x) = 1 - \frac{4}{(x-2)^2}$ (from A)

$\frac{x^2-4x+4-4}{(x-2)^2}$

$f' = \frac{x(x-4)}{(x-2)^2}$

$f''(x) = \frac{8}{(x-2)^3}$



$\frac{4}{-4} = -1$

$\frac{16}{2} = 8$

$\frac{36}{4} = 9$

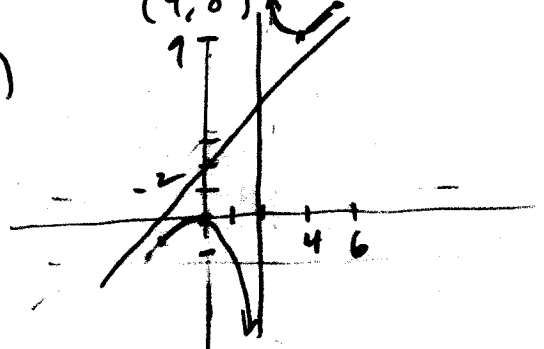
E) $(0, 0)$ nor f' test

$(-2, -1)$ min INC for int

$(6, 9)$ max INC to int.

$(4, 8)$ min f' test.

5)



IV $x \geq \ln(1+x)$

Let $f(x) = x - \ln(1+x)$

$f(0) = 0 - 0 = 0$

$f'(x) = 1 - \frac{1}{1+x}$

$= \frac{1+x-1}{1+x}$

$= \frac{x}{1+x} > 0 \; \forall x > 0$

So f increases from zero.

$\therefore \; x - \ln(1+x) > 0$

$x > \ln(1+x)$

QED

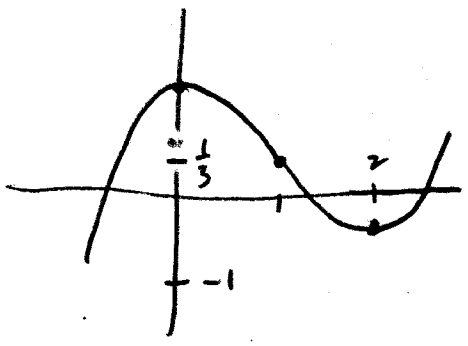
G)

f'	+	0	−	0 +
f''	−	0	+	

H) $(0,1)$ max f' ~~test~~

$(2, -\frac{1}{3})$ min f' ~~test~~

I)



EXTRA

$4x^2 - 12x + 5 = 0$

$(2x-1)(2x-5)$

$x = \{\frac{1}{2}, \frac{5}{2}\}$

$-2 \overset{\text{Set}}{=} \frac{1}{4p}$

$p = \frac{-1}{8}$

$y = -2x^2 + 6x - \frac{5}{2} =$

$-2(x^2 - 3x + \;\;) - \frac{5}{2}$

$-2(x - 3x + \frac{9}{4}) - \frac{5}{2} + \frac{9}{2}$

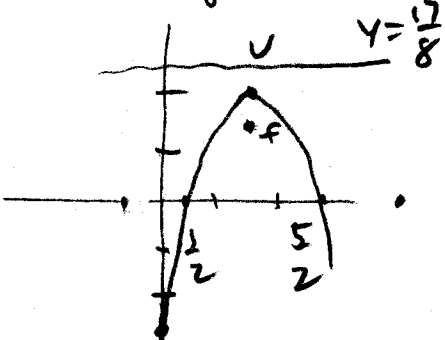
$y = -2(x - \frac{3}{2})^2 + 2$

$y|_{x=0} = -\frac{5}{2}$

vertex $(\frac{3}{2}, 2)$

focus $(\frac{3}{2}, \frac{15}{8})$

dir $y = \frac{17}{8}$



U $f(x) = Ax^3 + Bx^2 + Cx + D$

$(0,1) \; (1, \frac{1}{3})$

a) $f(0) \overset{\text{Set}}{=} D = 1 \; \therefore D = 1$

b) $f'(x) = 3Ax^2 + 2Bx + C$

$f''(x) = 6Ax + 2B$

c) $f'(0) \overset{\text{Set}}{=} 0 = C \; \therefore C = 0$

$f(x) = Ax^3 + Bx^2 + 1$

D) $f'(1) = A + B + 1 = \frac{1}{3} \quad f(1) = \frac{1}{3}$

E) $f''(1) = 6A + 2B = 0$

$3A + B = 0$ PI at $x = 1$

F) $A + B = \frac{-2}{3}$

$2A = \frac{2}{3}$

$A = \frac{1}{3} \; \therefore B = -1$

$f(x) = \frac{1}{3}x^3 - x^2 + 1$

$f'(x) = x^2 - 2x$

$x(x-2)$

$f''(x) = 2x - 2$

$2(x-1)$

$f'(1) = \frac{1}{3} - 1 + 1 = -\frac{1}{3}$

redo of 1990 Exam.

100 ↑	2
90 ↑	8
80 ↑	5
70 ↑	3
60 ↑	3
50 ↓	0

102

60

$\frac{1806}{21} = 86.$

Great Exam.

Possibly, Reward [II A)
to get them to divide in part c.
most did it any way in

Advanced Placement Calculus Name Calc Ch 6 mech

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Convince *me* that you understand the concept!

Chapter 6 "*Mechanical*" Exam

I Find the following limits.
Apply l'Hopital's rule at least once for each problem. (hint: both limits exist.) (10 pts ea)

a) $\lim_{x\to 0}\frac{e^x+e^{-x}-2}{1-\cos 2x}$

b) $\lim_{x\to 0}\frac{\sin x - x}{\tan x - x}$

II Some people say, "Rolle's Theorem is just a 'special case' of the Law of the Mean." Briefly and without using specific functions, explain Rolle's Thm and the Law of the Mean. Identify the "special case." What is the the "other name" for the Law of the Mean? (15 pts)

III Given: $f(x)=\frac{x^2}{x-2}$; $x \in [-2,6]$ (25 pts tot)

a) Rewrite $f(x)$ as a non-fraction and a fraction.
b) Find the domain of $f(x)$.
c) Write the equation(s) of any asymptotes.
d) Find $f'(x)$ and $f''(x)$ and draw appropriate number lines.
e) Identify and state the coordinates of all significant points (including intercepts). You will supply proper justification for all extrema, of course.
f) Draw the graph of $f(x)$ and its asymptotes on the same axis.

IV Given: $x \geq 0$. Prove: $x \geq \ln(1+x)$. (15 pts)

V Given: $f(x)=Ax^3+Bx^2+Cx+D$.
$f(x)$ has a relative maximum at $(0, 1)$ and a point of inflection at $\left(1,\frac{1}{3}\right)$. (25 pts tot)

a) What is the value of D?
b) Find $f'(x)$ and $f''(x)$.
c) What is the value of C?
d) Find $f(1)$ in terms of A and B.
e) Find $f''(1)$ in terms of A and B.
f) What are the values of A and B?
g) Draw properly labeled number lines for $f'(x)$ and $f''(x)$
h) What are the coordinates of the extrema?
i) Sketch $f(x)$.

Extra Credit ----------5 pts----------

Given: $y=-2x^2+6x-\frac{5}{2}$. Determine the equation of the directrix and the coordinates of the focus. Sketch. Label all significant items.

I $f(x) = x\cos x \quad x \in [-5, 5]$

$f = x\cos x$

$f' = \cos x - x\sin x$

$f'' = -\sin x - [\sin x + x\cos x]$

$= -2\sin x - x\cos x$.

$(-5, -1.42)$ min

$(-3.43, 3.29)$ max

$(-2.29, -1.51)$ PI

$(-.86, -.56)$ min

$(0, 0)$ PI

$(.86, .56)$ max

$(2.29, 1.51)$ PI

$(3.43, -3.29)$ min

$(5, 1.42)$ max

$(5, 1.42)$ max

f is odd

∴ only have to compute half of the parts.

The PI were extreme value of f' then $Y_1(x)$ to get range value

I used cTOOL $c = .72$

$f'(c) = .28$

II $\frac{x}{1+x^2} < \tan^{-1} x < x \quad \forall x > 0$

Let $f(x) = \frac{x}{1+x^2}$ $f(0) = 0$

$g(x) = \tan^{-1} x$ $g(0) = 0$

$h(x) = x$ $h(0) = 0$

$f'(x) = \frac{1+x^2 - (x)(2x)}{(1+x^2)^2}$ $g'(x) = \frac{1}{1+x^2}$

$= \frac{1-x^2}{(1+x^2)^2}$ $h'(x) = 1$

objective: since $f(0) = g(0) = h(0)$

if $f' < g' < h'$ then $f(a) < g(a) < h(a)$ for any $a > 0$

so need to show $f' < g' < h'$ so f is less than g is less than h

$f' \overset{?}{<} g'$ $g' \overset{?}{<} h'$

$\frac{1-x^2}{(1+x^2)^2} \overset{?}{<} \frac{1}{1+x^2}$ $\frac{1}{1+x^2} \overset{?}{<} 1$

$\frac{1-x^2}{1+x^2} < 1$ $1 < 1+x^2$

$1 - x^2 < 1 + x^2$ $0 < x^2$

True. True.

$\frac{x}{1+x^2} < \tan^{-1} x < x$

III Explain Ems

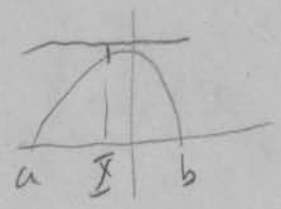
A) Rolle's Thm

f diff'd [a, b]

f(a) = f(b)

f' diff'd (a, b)

f'(x) = 0



used to prove f does not have 2 roots.

B) Intermediate Value Thm

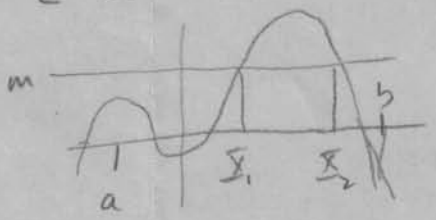
f cont [a, b]

If f(a) < m < f(b)

(or vice versa)

then There exists

c such that f(c) = m



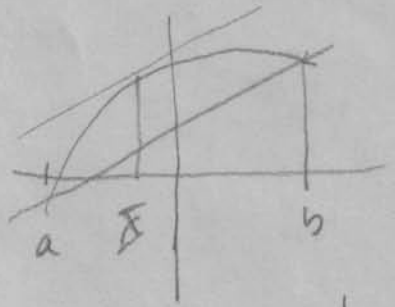
can be used to prove a fctn has at least 1 root

C) Law of the Mean (mean value Thm)

f def [a, b]

f' def (a, b)

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$



can determine when the instantaneous velocity was equal to average velocity.

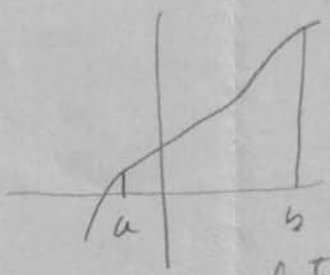
D) Corollary 3 to Law of Mean

f def [a, b]

f' def (a, b)

f' > 0

then f is an increasing fctn



could be used to show the fctn does not have another zero or to show one fctn is greater than less than another.

E) l'Hopital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)} = L$

Then $\lim_{x \to a} \frac{f'(x)}{g'(x)} = L$

no picture

$\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$ must be of these forms.

Greatly simplifies finding a limit in some situations

12/5/96

Erika Dangaiches 92

Hwa Shyong Lu 77

Advanced Placement Calculus Name Cal ch 6 mu

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Convince *me* that **you** understand the concept*!*

Chapter 6 Mechanical Make Up Exam

I Given: $f(x) = x \cos x$ and $x \in [-5, 5]$. (25 pts tot)

a) Determine (using your calculator) the coordinates and identify of all extrema and points of inflection. **Round all responses** to the nearest hundredth.
Note: You do *not* need to make a number line for this problem but you **do** need to describe your process.

b) Find the value of X, and $f'(X)$ where X is the X as guaranteed by Mean Value Thm.

II Prove: $\dfrac{x}{1+x^2} < \tan^{-1} x < x \quad \forall \; x > 0$ (25 pts)

III Explain 'Ems. Without selecting a specific function, explain the item and describe its features. Draw a sketch which illustrates those features. Finally, describe a situation where the item would be used.
(50 pts tot)

a) Rolle's Theorem

b) Intermediate Value Theorem

c) Law of the Mean (Mean Value Theorem)

d) Corollary 3 to Law of the Mean

e) l'Hopital's Rule

I a) $g(x) = \frac{1}{f(x)-1}$

$g'(x) = \frac{-1}{(f(x)-1)^2} \cdot f'(x)$

$(-)(+)(-1) > 0$

$\therefore g'(x) > 0$ so

g is increasing

b) $K(x) = h(f(x))$

$K'(x) = \underbrace{h'(f(x))}\, f'(x)$

$(+)(-1) > 0$

$\therefore$ K is also increasing fctn.

II $f(x) = \begin{cases} -x^2+x+5 & \text{if } x<0 \\ x^3-4x^2+x+5 & \text{if } x \geq 0 \end{cases}$ $x \in [-1,1]$

a) only questionable place is $x=0$ since f is a polynomial and all polynomials are continuous.

$f(0) = 5$ $\lim_{x\to 0^-} f(x) = 5 = \lim_{x\to 0^+} f(x)$ ②

= ③

f is continuous $\forall x$.

4 pts

Proof of Der

$\lim_{x\to 0^-} \frac{-x^2+x+5-5}{x}$

$\lim_{x\to 0^-} -x+1 = 1$ ←

$\lim_{x\to 0^+} \frac{x^3-4x^2+x+5-5}{x} =$

$\lim_{x\to 0^+} x^2-4x+1 = 1$ ←

$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$

Exists $\therefore$ f is Differentiable.

The MVT Does apply to this fctn. in This interval.

b) $f(-1) = -1+1+5 = 5$

$f(1) = 1-4+1+5 = 3$

$f'(x) = \begin{cases} -2x+1 & \text{if } x<0 \\ 3x^2-8x+1 & \text{if } x \geq 0 \end{cases}$

If $x<0$ set $-2x+1 = \frac{3-5}{1+1}$

$x = \frac{1}{2}$ Reject $x<0$

If $x>0$ set $3x^2-8x+1 = 0$

$x = \frac{8 \pm \sqrt{64-12}}{6}$

$\frac{8 \pm \sqrt{52}}{6}$

$\frac{8 \pm 2\sqrt{13}}{6}$

$\frac{4 \pm \sqrt{13}}{3}$

both of These are good answers

$X = \left\{ \frac{4+\sqrt{13}}{3} \right\}$.

p97

100	0
90	0
80	3
70	6
60	2
59	3

88

52

Probably too long.
change II as shown
eliminate I and
distribute those pts.
that should do it.

$$\frac{989}{14} = 70.6$$

Advanced Placement Calculus Name Calc Ch 6 mech

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Convince *me* that **you** understand the concept!

No Calculators!

Chapter 6 Mechanics Exam

I Let $f(x)$ be a function so that $f(x) < 1$ and $f'(x) < 0$ for all x. (15 pts)

a) Let $g(x) = \dfrac{1}{f(x) - 1}$. Prove that $g(x)$ is an increasing or decreasing function.

b) Let $h(x)$ be a differentiable function such that $h'(x) < 0$ for all x. Define $k(x) = h(f(x))$ Prove $k(x)$ is an increasing or decreasing function.

II Given: $f(x) = \begin{cases} -x^2 + x + 5 & \text{if } x < 0 \\ x^3 - 4x^2 + x + 5 & \text{if } x \geq 0 \end{cases}$ and $x \in [-1, 1]$ (20 pts)

a) Prove the *Mean Value Theorem* applies to $f(x)$ for the given interval (ie. Prove $f(x)$ is continuous in the closed interval and differentiable in the open interval.).

b) Find all X whose existence is guaranteed by the *Mean Value Theorem* for the given interval.

III Given: $f(x) = \dfrac{ax + c}{x^2 - b}$ and *i* $f(x)$ is an even function *ii* $\lim\limits_{x \to 2^+} f(x) = \infty$ and $x \in [-10, 10]$ (25 pts) *iii* $f'(1) = -2$

a) Determine the values of a, b, and c. (in that order)

b) Write the equations of any vertical or horizontal asymptotes.

c) Determine, identify, and justify the coordinates of all extrema.

IV Given $f(x) = \ln\left(\dfrac{x}{x - 1}\right)$ and $g(x) = \ln x - \ln(x - 1)$ (25 pts)

a) Determine the domain for $f(x)$. Determine the domain for $g(x)$.

b) Explain how you know that $f(x)$ and $g(x)$ are not equal functions.

c) Find $f'(x)$ and then evaluate it at $x = -1$.

d) Write an expression for $f^{-1}(x)$ (ie. the inverse function).

e) Demonstrate that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

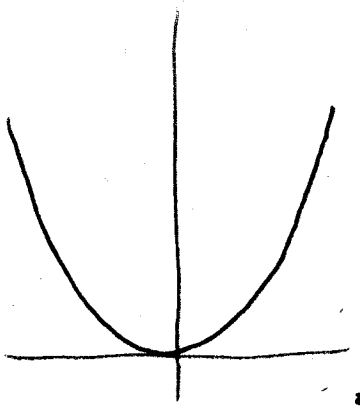
V Find the following limits (5 pts ea)

a) $\lim\limits_{x \to \pi/4} \dfrac{\sin(x - \pi/4)}{x^2 - \pi^2/16}$ b) $\lim\limits_{x \to 0} \dfrac{x - \tan x}{x - \sin x}$ c) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x}{\tan^{-1} 2x}$

Extra Credit ---------- **5 pts** ----------

You are driving along a highway at a constant speed of 55 mph. You observe a car one-half mile behind you. The car is moving fast and zooms past you exactly one minute later. How fast was the speeding car traveling if its speed was also constant?

I



$h(x) = x^2 + \cos x$

$h'(x) = 2x - \sin x$

$h''(x) = 2 - \cos x$

Since $|\cos x| \le 1$

$2 - \cos x > 0 \ \forall x$

so $h'' > 0 \therefore h$ is concave up $\forall x$.

II $f(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$ $\quad x^{\frac{2}{3}}(x+5)$ $\quad \sqrt[3]{4}\,(3)$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} + \frac{10}{3}x^{-\frac{1}{3}}$

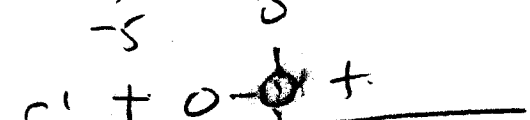
$\frac{5}{3}x^{\frac{2}{3}} + \frac{10}{3}\frac{1}{x^{\frac{1}{3}}}$

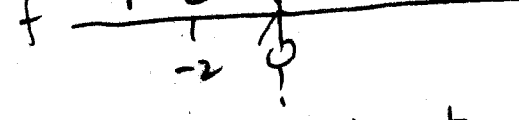
$\frac{5}{3}\left[\frac{x+2}{x^{\frac{1}{3}}}\right]$

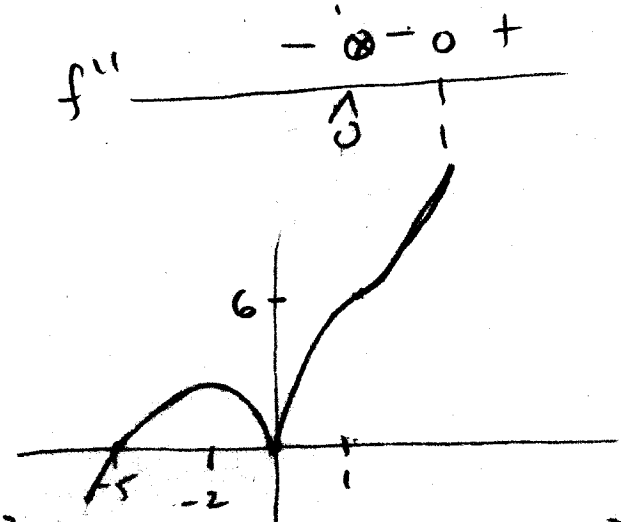
$f''(x) = \frac{10}{9}x^{-\frac{1}{3}} - \frac{10}{9}x^{-\frac{4}{3}}$

$\frac{10}{9}\left(\frac{1}{x^{\frac{1}{3}}} - \frac{1}{x^{\frac{4}{3}}}\right)$

$\frac{10}{9}\left(\frac{x-1}{x^{\frac{4}{3}}}\right)$







x-intercepts $(0,0)\ (-5,0)$ Rel max $(-2, 3\sqrt[3]{4})$ f increases $x<-2,\ x>0$

PI $(1,6)$ f decreases $-2<x<0$

f concave up $x>1$

f concave down $x<1,\ x\neq 0$

III a) $\lim_{x\to 0} \frac{1-\cos x}{x^2} \overset{0}{0}$

$\lim_{x\to 0} \frac{\sin x}{2x} \overset{0}{0}$

$\lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$

b) $\lim_{x\to 1} \frac{e^x - e}{5\ln x} \overset{1-1}{0.0} \overset{0}{0}$

$\lim_{x\to 1} \frac{e^x}{\frac{5}{x}} = \frac{e}{5}$

c) $\lim_{x\to 0^+} \sin x^{\ln x}$

$\ln y = \sin x \ln \sin x$

$= \frac{\ln \sin x}{\frac{1}{\ln x}} \overset{-\infty}{\infty}$

$\lim_{x\to 0}$

$= \frac{\frac{\cos x}{\sin x}}{-\frac{\cos x}{\sin^2 x}}$

d) $\lim_{x\to 0} x^{\frac{2}{\ln x}} \quad 0^0$

Let $y = x^{\frac{2}{\ln x}}$

$\ln y = \frac{2}{\ln x}\ln x$

$\ln y = 2$

$e^2 = y$

$\lim e^2 = e^2$

e) Not Ind.

Prob "d" is $0^0 = e^2$

Prob "c" is $0^0 = 1$

$\lim_{x\to 0}(\ln y = -\sin x)$

$\ln y = 0 \Rightarrow e^0 = y = 1$

Limit $= 1$

IV $f(x) = \ln x \quad g(x) = kx$

$f' = g' \quad \therefore \frac{1}{x} = k$

AND share common coordinate.

$\ln x \overset{\text{set}}{=} kx$

but $k = \frac{1}{x}$ so

$\ln x = \frac{1}{x} \cdot x$

$\ln x = 1$

$x = e$

So $k = \frac{1}{e}$

EXTRA

$Y_1 \quad f(x) = x + \cos x + 3$

$Y_2 \quad g(x) = 3x^3 - 2x + 4$

$Y_3 = Y_1 - Y_2$

2nd Calc.

~~abscissa~~ $-1.078, 0, .925$

Store Alpha → A STO ALPHA B.

$Y_1(A) = 2.394$

$(-1.078, 2.394) \quad (0, 4)$

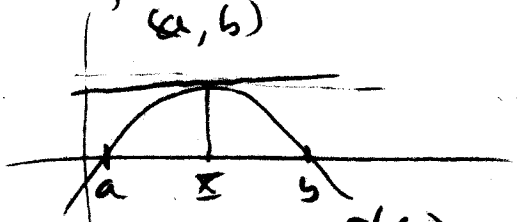
$Y_1(B) = 4.527$

$(.925, 4.527)$

V

A) Rolle's Thm

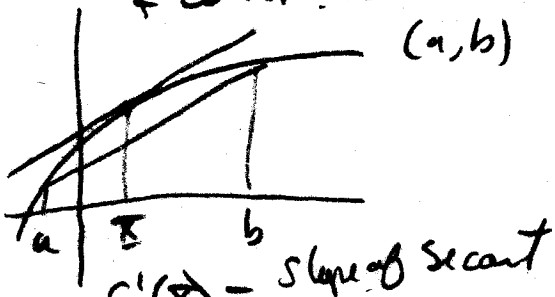
f cont & hadbl in (a, b)



$\bar{x} \in (a, b) \quad f'(\bar{x}) = 0$

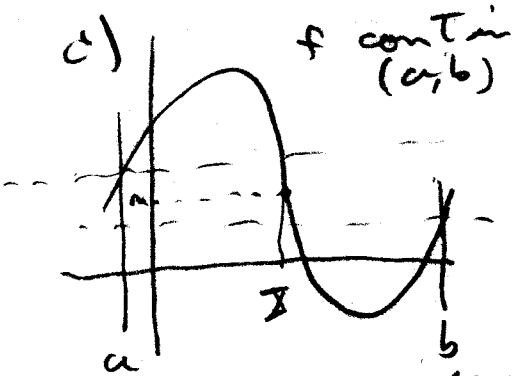
B) Law of The mean

f cont & has dv in (a, b)



$f'(\bar{x}) =$ slope of secant

C) f cont in (a, b)



$\bar{x} \in (a, b) \quad m \in (f(a), f(b))$

$f(\bar{x}) = m$

11/30/95

100		
90	2	
80	6	
70	5	
60	4	
59	3	67.1

91 ↓ 47

This is a Great exam.
the students are poor, very poor
with fractional exponents.
I can see no place which needs
a change. I guess there'll be 12
for which makeup. — which I
have to write ---
in my spare time....

11/23/98

100	0	
90	0	
80	3	
70	3	
60	7	
59	1	
	1077	
	15	
	71.8	

89 ↓ 52

I was out of class and
students didn't
step up and do their
job. Sloppy effort
on their part.

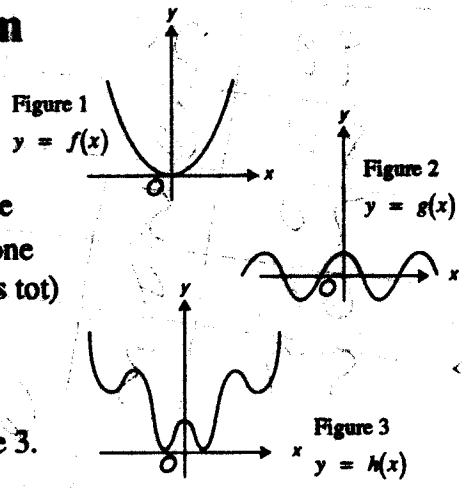
Advanced Placement Calculus Name Calc Ch6 mech

Copy original problem. Per _____ Date 11/30/95 98

Convince *me* that **you** understand the concept!

NO Decimals

Chapter 6 Mechanical Exam



I Let $f(x) = x^2$, $g(x) = \cos x$, and $h(x) = x^2 + \cos x$. From the graphs of f and g shown on the right in figure 1 and figure 2, one might think that the graph of h should look like figure 3. (15 pts tot)

a) Sketch the actual graph with $x \in [-6, 6]$ and $y \in [-6, 40]$

b) Use $h''(x)$ to explain why the graph of h does not look like figure 3.

II Given $f(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$ (30 pts tot)

a) Draw and fill in f, f', and f'' number lines.

b) Determine the coordinates of and identify *all* significant points.

c) Specifically state intervals where f is increasing, decreasing, concave up, and concave down.

d) Sketch $f(x)$

III Find the following limits (all limits exist) (5 pts ea)

a) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ b) $\lim_{x \to 1} \frac{e^x - e}{5 \ln x}$ c) $\lim_{x \to 0^+} (\sin x)^{\sin x}$ d) $\lim_{x \to 0^+} x^{\frac{2}{\ln x}}$

e) Agree or disagree. Support your position. "Any number to the zero power is one."

IV Given $f(x) = \ln x$ and $g(x) = kx$. Determine k so that g is tangent to f. (10 pts)

V Define each of the following. For each, draw a sketch illustrating your definition. Be sure to indicate all points mentioned. Do not use specific functions nor numbers. (20 pts tot)

OR

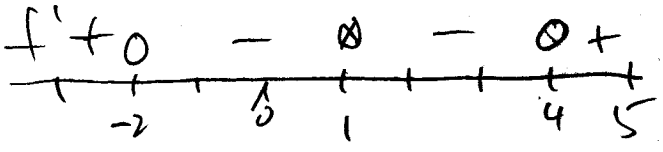
a) Rolle's Therorem b) Law of the Mean (Mean Value Theorem) c) Intermediate Value Theorem

Extra Credit ---------- 5 pts ----------

Let $f(x) = x + \cos x + 3$ and $g(x) = 3x^3 - 2x + 4$. Determine the coordinates (accurate to 3 places) of the points of intersection of f and g

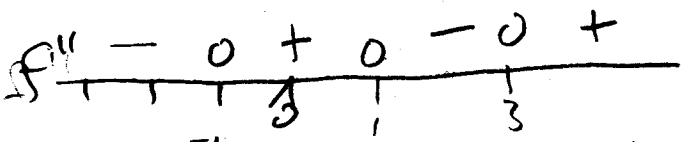
Replace

(I) No end pts ~5



a) f has rel max $x=-2$ f' test
$x=5$ END PT INC to it

b) f has rel min $x=-3$ END PT INC fm it
$x=4$ f' test.

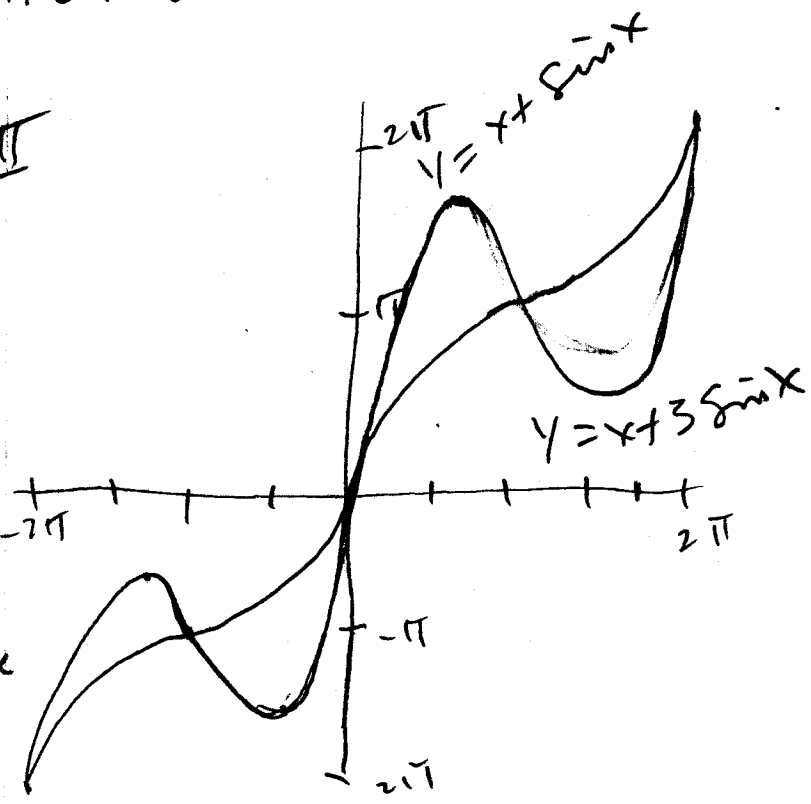


c) Note: when f' is decreasing $f'' \le 0$ and f is concave up.

f is concave up $-1 < x < 1$ and $3 < x < 5$

d) $f(1) = 0$

II



b) $y = x + b\sin x = f(x)$
a line is Tangent when it has the Same slope AND shares the same coordinate

$y' = 1 \quad f' = 1 + b\cos x$

$1 + b\cos x \overset{\text{Set}}{=} 1$

$b\cos x = 0$

$x \in \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Same Slope

Same Coord

$x + b \overset{\text{Set}}{=} x + b\sin x$

$b - b\sin x = 0$

$b(1 - \sin x) = 0$

Both True for $x \in \left\{-\frac{3\pi}{2}, \frac{\pi}{2}\right\}$

$x \in \left\{-\frac{3\pi}{2}, \frac{\pi}{2}\right\}$

Extra

$f(x) = \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}}$

$f'(x) = (1+x)^{\frac{3}{2}} - (1+x)^{\frac{1}{2}}$

$= (1+x)^{\frac{1}{2}}(1+x-1)$

$= x(1+x)^{\frac{1}{2}}$

$g(x) = \frac{2}{3}(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}}$

$g'(x) = \frac{2}{3}(1+x)^{\frac{3}{2}} + x(1+x)^{\frac{1}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}}$

$= x(1+x)^{\frac{1}{2}}$

$f - g = \frac{(1+x)^{\frac{3}{2}}}{15}\left(6(1+x) - 10 - 10x + 4(1+x)\right)$

$= \frac{(1+x)^{\frac{3}{2}}}{15}(6 + 6x - 10 - 10x + 4 + 4x)$

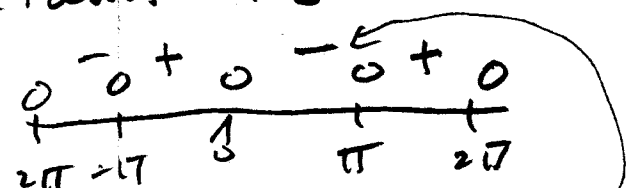
zero.

$f = g$ or differs by at most a constant.

They don't differ, the constant is zero.

II c) a point is an extreme value if it is a critical value ($f'(x)=0$), an end pt or a place where the fcn is continuous but the der DNE (cusp).
The points in Part b are not critical values since $f(-\frac{3\pi}{2}) = f'(\frac{\pi}{2}) = 1$. The der exists there and they are not endpoints.
Conclusion: The points in b are NOT rel max.

d) $f(x) = x + b\sin x$
$f'(x) = 1 + b\cos x$
$f''(x) = -b\sin x \overset{\text{set}}{=} 0$
$x = \{k\pi\}$



$f''(\frac{\pi}{2}) = -b(+1) < 0$

f has PI where f changes concavity. This occurs when f' changes signs. f has PI at $-\pi, 0, \pi$ because f changes concavity at those x values.

$f(-\pi) = -\pi + 0 = -\pi$ $(-\pi, -\pi)$
$f(0) = 0 + 0 = 0$ $(0, 0)$
$f(\pi) = \pi + 0 = \pi$ (π, π)

The coord of the PI also satisfy $y=x$. There are no other PI so none lie off of $y=x$.

III

a) $\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x - e^{-x}}{2\sin 2x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x \times e^{-x}}{4\cos 2x} = \frac{1+1}{4} = \frac{1}{2}$

b) $\lim_{x\to \frac{\pi}{2}^-} (2x - \pi)\sec x$

$\lim_{x\to \frac{\pi}{2}^-} \frac{2x-\pi}{\cos x}$ $\frac{0}{0}$

$\lim_{x\to \frac{\pi}{2}^-} \frac{2}{-\sin x} = -2$

c) $\lim_{x\to 0^+} \left(\frac{1}{e^x - 1} - \frac{1}{x}\right)$

$\lim_{x\to 0^+} \left[\frac{x - e^x + 1}{x(e^x - 1)}\right]$ $\frac{0}{0}$

$\lim_{x\to 0^+} \frac{1 - e^x}{e^x - 1 + xe^x}$ $\frac{0}{0}$

$\lim_{x\to 0^+} \frac{-e^x}{e^x + e^x + xe^x}$

$\lim_{x\to 0^+} \frac{-1}{2 + x} = -\frac{1}{2}$

11/19/99

100	0
90	2
80	9
70	7
60	3
59	2

97

40

1749
23

76.0

Include the "suggestions" listed for Part II a).

Generally, an excellent exam!

12-4-97

Oliver Fernandez	95
Ken Tsao	85
Matt Dagenais	80
Rebecca Wells	76
Mike Evans	71
Ian Spradlin	68
Trevor Hughes	41
Troy Forte	38

Advanced Placement Calculus

Name Calc Ch6

Per _____ Date Mechanical

Copy original problem.

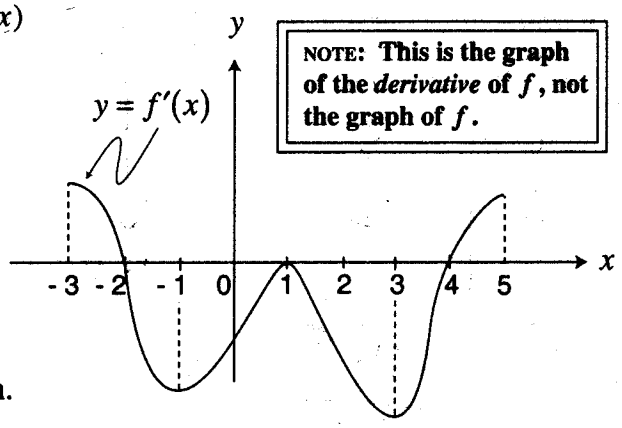
Convince *me* that **you** understand the concept!

99

Chapter 6 Mechanical Exam

I

The figure shows the graph of $f'(x)$, the derivative of a function $f(x)$. The domain of $f(x)$ is the set of all real numbers x such that $-3 \le x \le 5$. (40 pts tot)



a) For what values of x does f have a relative maximum? Explain.

b) For what values of x does f have a relative minimum? Explain.

c) On what intervals is the graph of f concave upward? Use $f'(x)$ in your justification.

d) Suppose that $f(1) = 0$. On an xy-plane, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

II

This problems deals with functions defined by $f(x) = x + b\sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$. (40 pts tot)

use your grapher to assist you to

a) Sketch the graphs of two of these functions: $y = x + \sin x$ and $y = x + 3\sin x$ on the same axis.

b) Find the x-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x$.

c) Are the points of tangency described in part (b) relative maximum points of f? Why?

d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

III

Find the following limits. (All exist). Use l'hospital's rule at least once. Be sure your reasoning is very clear. (20 pts tot)

a) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos 2x}$

b) $\lim_{x \uparrow \pi/2} (2x - \pi)\sec x$

c) $\lim_{x \downarrow 0} \left(\dfrac{1}{e^x - 1} - \dfrac{1}{x} \right)$

Extra Credit ---------- 5 pts ----------

Given: $f(x) = \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}}$ and $g(x) = \frac{2x}{3}(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}}$.

Find $f'(x)$ and $g'(x)$. What do your answers suggest? What is that constant?

Calculus — Graphs of Composite Functions

10/27/77

Given: $f(x)=\sqrt{1-x}$, $x\le 1$; $g(x)=\sqrt{x}$, $x\ge 0$.

a) what is D_f?
b) what is D_g?
c) what is D_h.
d) Draw the graph of
e) Draw graph of

Define $h(x)=f(g(x))=\sqrt{1-\sqrt{x}}$ $x\ge 0$, $\sqrt{x}\le 1$.

$D_h=\{x \mid 0\le x\le 1\}$. describe the concavity of $h(x)$.

Fig. 1

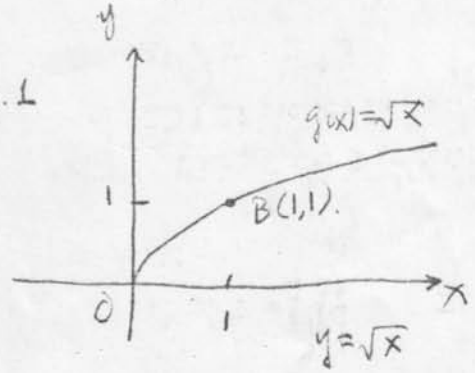
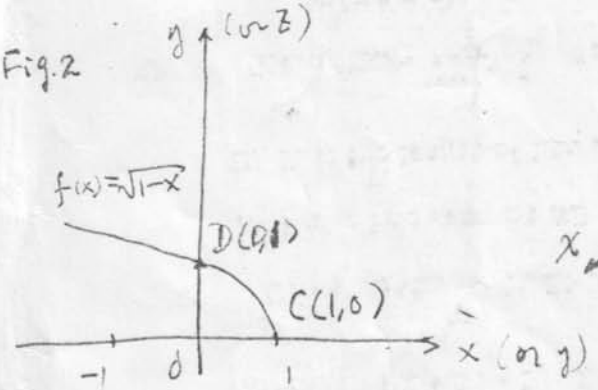


Fig. 2



$z=\sqrt{1-y}$

Fig. 3

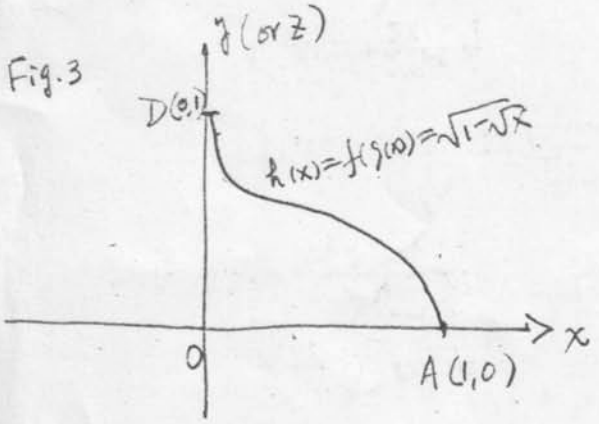
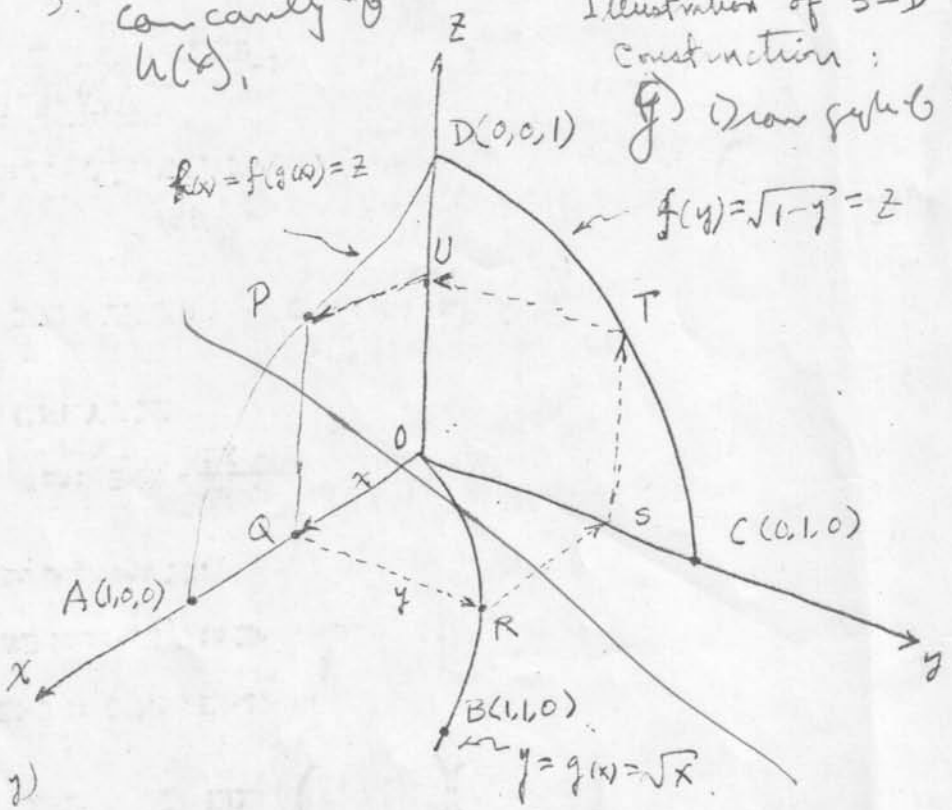


Illustration of 3-D Construction:

g) Draw graph of h



Use the 3-D Construction method to graph $y=x^2$, $z=\sin y$.

$\therefore z=\sin x^2$.

Calc: ch 6 meth

$$f(x) = \frac{64}{\sin x} + \frac{27}{\cos x}$$

$$f'(x) = -\frac{64\cos x}{\sin^2 x} + \frac{27\sin x}{\cos^2 x}$$

$$\frac{-64\cos^3 + 27\sin^3 x}{\sin^2 x \cos^2 x} \overset{\text{set}}{=} 0$$

$$27\sin^3 x = 64\cos^3 x$$

$f'\left(\frac{\pi}{4}\right) < 0$

$$\tan^3 x = \frac{64}{27}$$

$$\tan x = \frac{4}{3}$$

$$- \quad 0 \quad +$$

$0 \quad \frac{\pi}{4} \quad 1 \quad 1$

.927

Show that

$$f(x) = \frac{64}{\sin x} + \frac{27}{\cos x}$$

has a minimum value but no maximum value on the interval $(0, \frac{\pi}{2})$

Len Calc
ch 6 mech

"Four Corners of the World"

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

1. Find the value c which satisfies the Mean Value Theorem for $f(x) = \ln x$ over $1 \le x \le 4$.

   a) $\frac{\ln 4}{3}$ b) $\frac{3}{\ln 4}$ c) $\ln 4$ d) $3 \ln 4$ e) $\frac{4}{3}$

2. If $y = \sqrt{x}^{\sqrt{x}}$ then $\ln y =$

   a) $\frac{x \ln x}{2}$ b) $\frac{\sqrt{x} \ln x}{2}$ c) $\frac{x \ln x}{4}$ d) $\frac{\ln x}{4}$ e) $\sqrt{x} \ln x$

3. If $y = \cos^2 x - \sin^2 x$ then $\frac{d^2y}{dx^2} =$

   a) 0 b) $4 \cos x \sin x$ c) $-2 \sin 2x$ d) $4 \sin 2x$ e) $-4 \cos 2x$

4. If $y = 3x^{100}$ then $\frac{d^{100}y}{dx^{100}} =$

   a) $3(100!)$ b) $3(100!)x$ c) 0 d) $300x^{99}$ e) $300!$

5. If $f(x) = \sqrt{x^3}$ then $f'''\left(\frac{1}{4}\right) =$

   a) -3 b) $\frac{-3}{64}$ c) $\frac{9}{64}$ d) $\frac{9}{32}$ e) $\frac{3}{8}$

6. If $f(x) = (\sin x)^2$, then $\frac{d^2y}{dx^2} =$

   a) $2 \cos x$ b) $\cos 2x$ c) $\sin^2 x$ d) $\cos^2 x$ e) $2 \cos 2x$

7. If $f(x) = \frac{1}{x}$, then the sixth derivative of $f(x)$ is:

   a) $\frac{6}{x^6}$ b) $\frac{6!}{x^7}$ c) $\frac{1}{x^6}$ d) $-\frac{6}{x^7}$ e) 0

8. The absolute minimum value of $f(x) = \frac{3x-1}{x+2}$ on $[0,2]$ is:

   a) 0 b) 2 c) $-\frac{1}{2}$ d) $\frac{5}{4}$ e) -2

9. The x-coordinate of the absolute maximum of $f(x) = 3x^3 - \frac{9}{2}x^2$ on $[-2,2]$ is:

   a) 0 b) 2 c) 1 d) -2 e) $\frac{1}{2}$

10. Find the absolute maximum value of $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-1,2]$

    a) 19 b) 14 c) -6 d) 5 e) 21

11. $\lim_{x\to\infty}\left(3+2e^{x}\right)^{3/x}=$

a) 3 b) $3e$ c) 0 d) e^3 e) Nonexistent

12. For the function $f(x)=|x-4|^2$, find the value of $f''(c)$ where c is a critical number of the function.

a) 0 b) 2 c) 4 d) 3 e) $f''(c)$ does not exist

13. Find all values of x where $f(x)$ is increasing if $f(x)=\left(2x^2-8\right)^{1/3}$.

a) $(-2,0)\cup(2,\infty)$ b) $(-\infty,-2)\cup(2,\infty)$ c) $(-\infty,0)\cup(2,\infty)$ d) $(-2,2)$ e) $(0,\infty)$

14. For $f(x)=x^3+ax^2$ to have a relative minimum at $x=1$ then $a=$

a) $-\frac{3}{2}$ b) -1 c) $-\frac{2}{3}$ d) $\frac{2}{3}$ e) 1

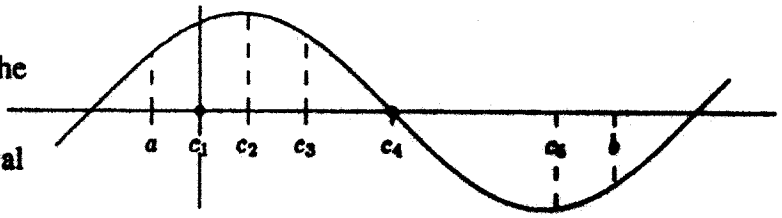
15. The greatest slope of $f(x)=x^3-9x^2$ for $0\le x\le 10$ occurs at $x=$

a) 0 b) 2 c) 3 d) 6 e) 10

16. If $f(x)=x^{1/3}-x^{2/3}$ then f has relative extrema points at:

a) $x=0$ only b) $x=\frac{1}{8}$ only c) $x=0$ and $x=\frac{1}{8}$ d) $x=\frac{1}{2}$ e) $x=0$ and $x=\frac{1}{2}$

17. Which value of c_i, most closely approximates the value of c satisfying the conclusion of the Mean Value Theorem for the function f over the interval $a\le x\le b$.



a) c_1 b) c_2 c) c_3 d) c_4 e) c_5

18. For $f(x)=x^{2/3}-1$ on $[-1,1]$ the "c" value that satisfies Rolle's Theorem is:

a) -1 b) 1 c) 1 or -1

d) There is no c value because $f(-1)$ is not zero.

e) There is no c value because f is not differentiable on $(-1,1)$.

19) For the following functions

I. $f(x)=x^3+1$ on $[-1,1]$ II. $f(x)=1-|x|$ on $[-1,1]$ III. $f(x)=x^2-x$ on $[0,1]$

Rolle's Theorem holds for :

a) I only b) II only c) III only d) I and II only e) I and III only

20) The Mean Value Theorem does not apply to $f(x)=|3x-2|$ on $[0,1]$ because:

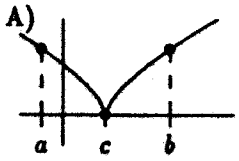
a) $f(0)\neq f(1)$ b) $f(0)>f(1)$

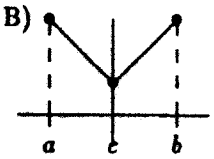
c) $f(x)$ is not continuous on $[0,1]$.

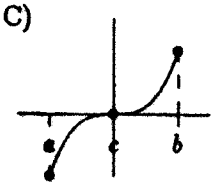
d) $f(x)$ is not differentiable on $(0,1)$.

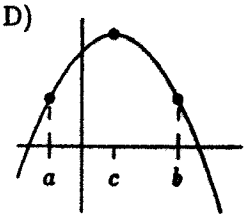
e) $\lim_{x\to 2/3} f(x)=f(\frac{2}{3})$

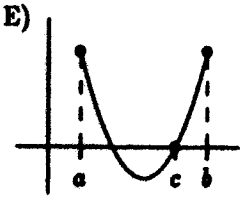
21. Which picture meets the conditions of Rolle's Theorem over an interval [a, b] and which indicates a value c meeting the conclusion of Rolle's Theorem.

A) 

B) 

C) 

D) 

E) 

22. Find the number(s) for "c" which satisfy the conditions of the Mean Value Theorem of differential calculus for $f(x) = x^3 - 2x^2$ on $[0, 2]$.

a) 0 b) $\frac{4}{3}$ c) $0, \frac{4}{3}$ d) $\frac{4}{3}, 2$ e) $\frac{2}{3}$

23. Given $f(x) = -\frac{1}{2}x^4 + \frac{1}{3}x^3 + 3x^2 - 4$. Find the intervals on which f is increasing.

a) $-\frac{3}{2} < x < 0$ or $x > 2$ b) $x < -\frac{3}{2}$ or $x > 2$ c) $-\frac{3}{2} < x < 2$
d) $x < -\frac{3}{2}$ or $0 < x < 2$ e) $x > 2$

24. Given $H'(x) = (x-1)(x+2)(x-3)^2$. Determine the x coordinates of the relative maximum points of $H(x)$.

a) 1 b) -2 c) 3 d) $1, 3$ e) $-2, 3$

25. Let $f(x) = 3bx^2 + ax - 5$ and let $(2, -3)$ be a relative extremum of f, then the values of a and b respectively are:

a) $2, -3$ b) $2, -6$ c) $2, -\frac{1}{6}$ d) $-6, 2$ e) $-2, 6$

26. The total number of relative extrema of the function H whose derivative for all x is given by $H'(x) = 3x(x-5)(2x-3)^2(3x-4)^4$ is:

a) 0 b) 1 c) 2 d) 3 e) 4

27. If $f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } -5 \le x \le 5 \\ x - 5 & \text{if } x > 5 \\ -x - 5 & \text{if } x < -5 \end{cases}$ then f is increasing on

a) $(5, \infty)$ b) $(-\infty, \infty)$ c) $(-5, 0) \cup (5, \infty)$ d) $(-\infty, -5) \cup (0, 5)$ e) $(-5, 5)$

28. If $f(x) = \begin{cases} 4 - x^2 & \text{if } |x| < 2 \\ |x| - 2 & \text{if } |x| \ge 2 \end{cases}$ then f has minimum values when $x =$

a) 0 b) 2 c) 1 or -1 d) 2 or -2 e) -2

29) The equation of the tangent to the curve $y = x^3 - x^2 + 2x$ at its point of inflection is:

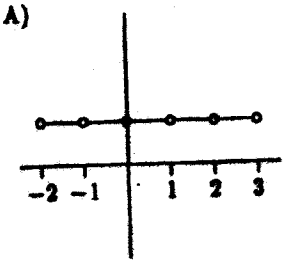
a) $y = 2x$ b) $23x - y = 45$ c) $45x - 27y = -1$ d) $9x - 27y = -13$ e) $63x - 27y = 5$

30) If $f(x) = 3x^5 - 10x^4 + 7x - 5$, then there are inflection points when $x =$

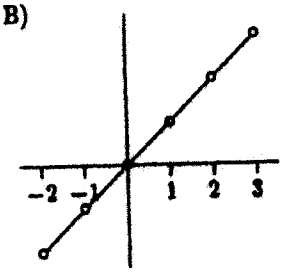
a) 0 b) 2 c) 0 or 2 d) -2 e) 0 or -2

31. Which graph of f best suits the description of f over $-2 \le x \le 3$.
$f'(n)$ is undefined for n any natural number.
$f'(x) = 1$ for any $x \neq n$.

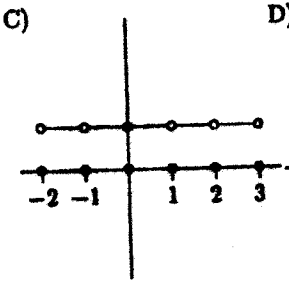
A)



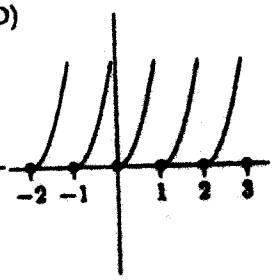
B)



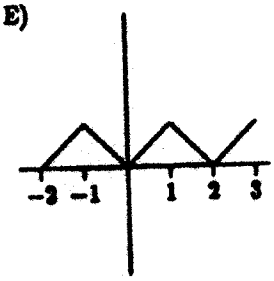
C)



D)

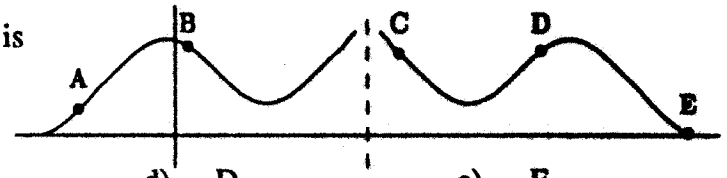


E)



32. At which of the five points on the graph on the right is

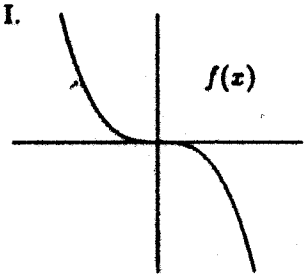
$$\frac{dy}{dx} > 0 \text{ and } \frac{d^2y}{dx^2} < 0$$



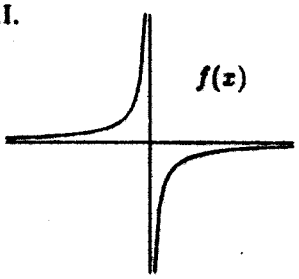
a) A b) B c) C d) D e) E

33. For which of the following functions is $f'(x) < 0$ whenever $f(x)$ defined?

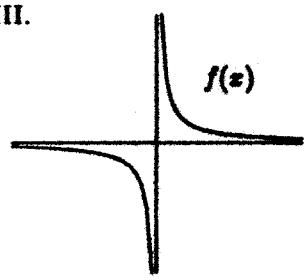
I.



II.

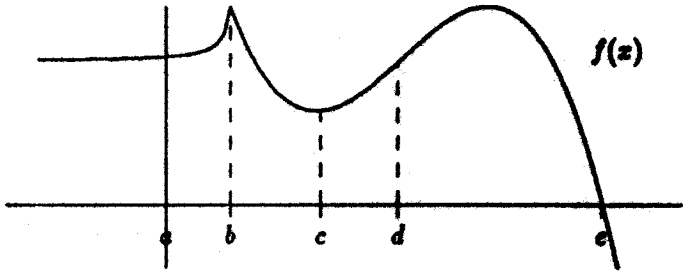


III.



a) I only b) II only c) III only d) I and II only e) I and III only

34. On the graph of f shown on the right, at which indicated x-value does it appear that the rate of change in the slope of f is zero? At $x =$



a) A b) B c) C d) D e) E

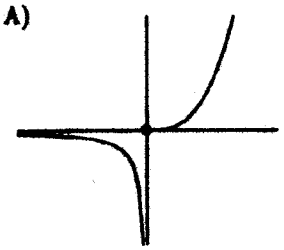
35. If $f(x) = x^4 + 4x^3 + 6x^2 + 4x$, the inflection point(s) is(are):

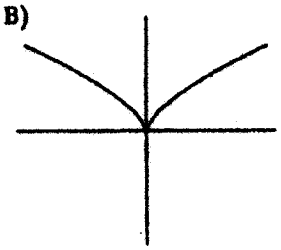
a) $(-1, 0)$ b) $(1, 16)$ c) $(0, 1)$ d) There are none. e) $(1, 16)$ and $(-1, 0)$

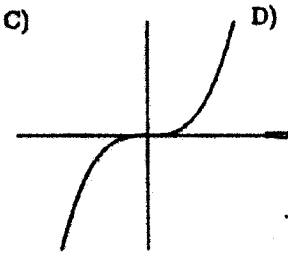
36. If $f(x) = \frac{x^2 - 1}{x^2}$, then the graph of f is concave up for:

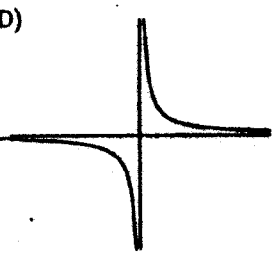
a) All values of x b) $-1 < x < 1$ c) $x < -1$ or $x > 1$ d) No values of x e) $-\sqrt{2} < x < \sqrt{2}$

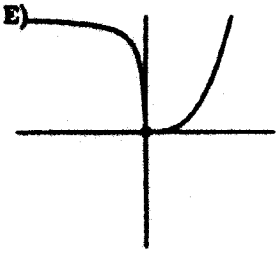
37. Which graph of f suits the description of f, a continuous function, where
$f(x)>0,\ f'(x)>0,\ f''(x)>0$ for $x>0$ **and** $f(x)>0,\ f'(x)<0,\ f''(x)<0$ for $x<0$.

A) 

B) 

C) 

D) 

E) 

38. If f is a continuous function with $f''(x)=-5(3+2x)(2-3x)^2(3x-1)^3$, find the set of values of x for which f has an inflection point.

a) $\left\{-\frac{3}{2},\frac{1}{3},\frac{2}{3}\right\}$ b) $\left\{0,-\frac{3}{2},\frac{1}{3},\frac{2}{3}\right\}$ c) $\left\{-\frac{3}{2},\frac{1}{3}\right\}$ d) $\left\{-\frac{3}{2}\right\}$ e) $\left\{\frac{1}{3}\right\}$

39) Let $f(x)=2x^3+3x^2-36x$. Which of these choices is necessary to makethe point $(2,-44)$ is a minimum?

I. $f'(2)=0$ II. $f''(2)<0$ III. $f(2)<0$

a) I only b) II only c) III only d) I and II only e) **I, II, and III**

40. You know the relative minimum of $g(x)=\dfrac{3x^2+3}{x}$ is at $x=1$ because:

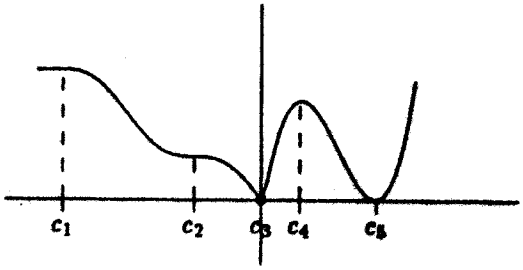
I. $g'(1)=0$ II. $g''(1)>0$ III. $g(1)=6$

a) I only b) II only c) III only d) I and II only e) **I, II, and III**

41. If $f(x)=\dfrac{x^2+1}{x}$, then f has a relative minimum when $x=$

a) 0 b) 1 c) -1 d) no minimum e) $\sqrt{3}$

42. On the graph, which indicated x-value would satisfy: $f''(x)>0$ when $f'(x)=0$?



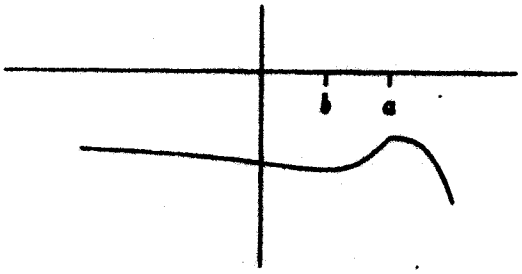
a) c_1 b) c_2 c) c_3 d) c_4 e) c_5

43. Given the graph of f, which of the following statements are true?

I. $f''(x)<0$ when $x>a$.

II. $f'(x)<0$ when $x>b$.

III. $f'(a)=0$.



a) I only b) II only c) III only d) I and II only e) **I, II, and III**

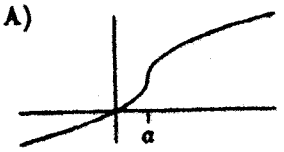
44. For the function $f(x) = \sqrt[3]{x}$, find $\lim_{x \to \infty} f'(x)$.

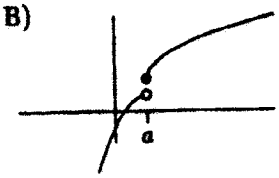
a) ∞ b) 0 c) 1 d) 3 e) 2

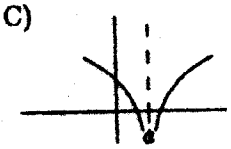
45. The function $f(x) = \dfrac{x^3 - 2x + 1}{(1-x)^2}$ has an oblique asymptote:

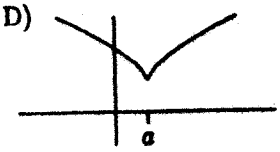
a) $y = x$ b) $y = x - 2$ c) $y = x + 2$ d) $y = 2 - x$ e) $y = x - 3$

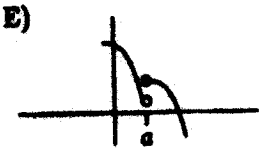
46. If f is continuous $\forall$ x and differentiable for all x except $x = a$ and always concave downward, then which of the following could be a graph for f?

A) 

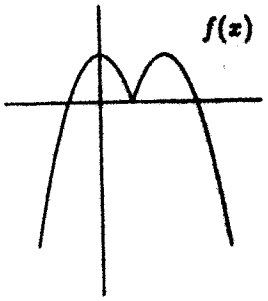
B) 

C) 

D) 

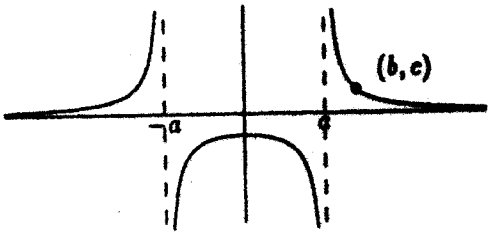
E) 

47. For the graph of f, which of the following statements is true?



a) $f(x)$ is continuous.
b) $f(x)$ is differentiable for all x.
c) $f(x) = |f(x)|$
d) $f(x)$ is an even function.
e) $f(x)$ has no relative minimum point.

48. For the function f with the given graph, which of the following statements is not true?

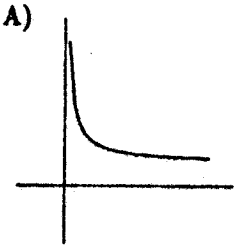


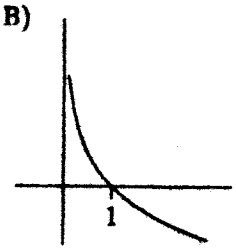
a) $f(x)$ is discontinuous at $-a$ and a.
b) $f'(x)$ is positive if $x < 0$.
c) There is a relative maximum at $x = 0$.
d) $f''(x)$ is positive if $-a < x < a$.
e) $\lim_{x \to -b} f(x) = c$.

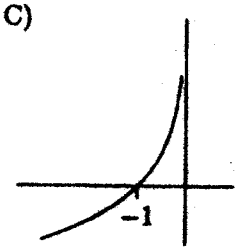
49. $\lim_{x \to \infty} \ln\left(1 + \dfrac{1}{x}\right)^{2x} =$

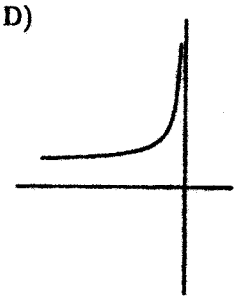
a) 2 b) ∞ c) 1 d) $2e$ e) e^2

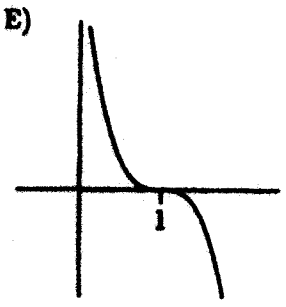
50., The graph of $y = \ln\left(\dfrac{1}{x}\right)$ is :

A) 

B) 

C) 

D) 

E) 

51. $\lim_{x \to \pi/4} \frac{\sin\left(x - \frac{\pi}{4}\right)}{x^2 - \pi^2/16} =$

a) 0 b) $\frac{2}{\pi}$ c) $\frac{1}{8\pi}$ d) $\frac{8}{3\pi}$ e) **Nonexistent**

52. $\lim_{x \to \infty} \frac{\ln\left(x^3\right)}{e^{2x}} =$

a) 0 b) ∞ c) 1 d) $\frac{1}{2}$ e) $\frac{3}{2}$

53. In the invalid argument:

$$\lim_{x \to 0} \frac{x^6 - x^5 + x^4 - x^3}{x^5 - x^4 + x^3 - x^2} = \lim_{x \to 0} \frac{6x^5 - 5x^4 + 4x^3 - 3x^2}{5x^4 - 4x^3 + 3x^2 - 2x} \quad \textbf{Step 1}$$

$$\lim_{x \to 0} \frac{30x^4 - 20x^3 + 12x^2 - 6x}{20x^3 - 12x^2 + 6x - 2} \quad \textbf{Step 2}$$

$$\lim_{x \to 0} \frac{120x^3 - 60x^2 + 24x - 6}{60x^2 - 24x + 6} \quad \textbf{Step 3}$$

$$\lim_{x \to 0} \frac{360x^2 - 120x + 24}{120x - 24} \quad \textbf{Step 4}$$

$$\lim_{x \to 0} \frac{720x - 120}{120} \quad \textbf{Step 5}$$

At what step should one stop using L'Hôpital's Rule and evaluate the limit? Step:

a) 1 b) 2 c) 3 d) 4 e) 5

54. Evaluate $\lim_{x \to \frac{\pi}{2}^-} \left(\frac{\sin^2 x}{1 - \cos x}\right)^{\tan x}$

a) 0 b) 1 c) $\frac{\pi}{2}$ d) ∞ e) e

55. Evaluate $\lim_{x \to 0} \frac{x - \tan x}{\sin x - x}$

a) 2 b) 0 c) ∞ d) 1 e) $\frac{1}{2}$

56. Evaluate $\lim_{x \to \infty} \frac{e^x}{x^2 + 4}$

a) There is no limit b) $\frac{1}{4}$ c) 0 d) 1 e) $\frac{1}{2}$

57. $\lim_{x \to 0} \dfrac{\sin^{-1} x}{\tan^{-1} 2x}$

a) ∞ b) $-\infty$ c) 0 d) 1 e) $\frac{1}{2}$

58. $\lim_{x \to \infty} x^{1/x} =$

a) ∞ b) 1 c) e d) $\frac{1}{e}$ e) 0

59. Evaluate $\lim_{x \uparrow \pi/2} \dfrac{\sec x}{\tan x}$.

a) ∞ b) $-\infty$ c) 1 d) -1 e) 0

60. Evaluate $\lim_{x \to \infty} [\ln x - \ln(1+x)]$.

a) There is no limit. b) $-\infty$ c) -2 d) 0 e) 1

61. $\lim_{x \to \infty} \left(\sqrt{x^2 + 2x} - x\right) =$

a) 0 b) $\frac{1}{2}$ c) 1 d) 2 e) **The limit does not exist.**

62. $D_x\left[\ln \sec^{-1} x^2\right] =$

a) $\dfrac{1}{\sec^{-1} x^2}$ b) $\dfrac{2x}{\sec^{-1} x^2}$

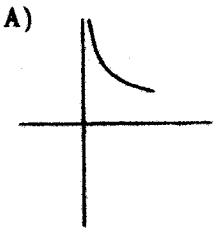
c) $\dfrac{1}{\left(\sec^{-1} x^2\right)\left(x^2\sqrt{x^4 - 1}\right)}$ d) $\dfrac{2}{x\sqrt{x^2 - 1}\left(\sec^{-1} x^2\right)}$ e) $\dfrac{2}{x\sqrt{x^4 - 1}\left(\sec^{-1} x^2\right)}$

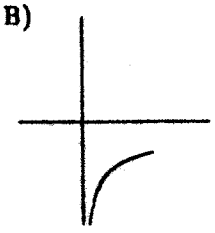
63. $\dfrac{d}{dx}\left[\tan 3^{2x}\right] =$

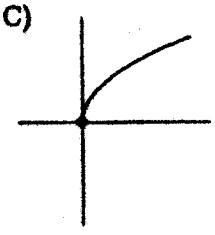
a) $\sec^2 3^{2x}$ b) $\sec^2 3^{2x} \ln 3$ c) $\left(\sec^2 3^{2x}\right)3^{2x}$

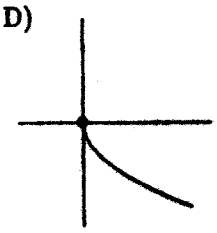
d) $2\left(3^{2x}\right)\left(\sec^2 3^{2x}\right)\ln 3$ e) $\left(3^{2x}\right)\left(\sec^2 3^{2x}\right)\ln 3$

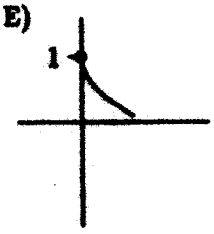
64. $\lim_{x \to 0^+} \dfrac{\ln x}{x}$ behaves graphically near zero like:

A) 

B) 

C) 

D) 

E) 

I $f(x) = \begin{cases} -x^2+x+5 & \text{if } x<0 \\ x^3-4x^2+x+5 & \text{if } x\geq 0 \end{cases}$ $x\in[-1,1]$

A) Only questionable place is $x=0$ since f is a polynomial and all polynomials are continuous.

$f(0) = 5$ ①

$\lim_{x\to 0^-} -x^2+x+5 = 5$ ②

$\lim_{x\to 0^+} x^3-4x^2+x+5 = 5$ ③

① = ② = ③

f is continuous everywhere

Next Does $f'(x)$ exist in $(-1,1)$? again, only place to check is at $x=0$

$\lim_{x\to 0^-} \frac{-x^2+x+5-5}{x-0}$

$\lim_{x\to 0^-} -x+1 = 1$

$\lim_{x\to 0^+} \frac{x^3-4x^2+x+5-5}{x}$

$\lim_{x\to 0^+} x^2-4x+1 = 1$

$=$ so f' exists in this interval

$=$ MVT Does apply to this fctn.

B) $f'(x) = \begin{cases} -2x+1 & \text{if } x<0 \\ 3x^2-8x+1 & \text{if } x\geq 0 \end{cases}$

$f(-1) = -1-1+5 = 3$

$f(1) = 1-4+1+5 = 3$

If $-1\leq x<0$

$-2x+1 \overset{\text{set}}{=} \frac{3-3}{1+1}$

$-2x+1 = 0$

$-2x = -1$

$x = \frac{1}{2}$ Reject.

If $0\leq x\leq 1$

$3x^2-8x+1 \overset{\text{set}}{=} 0$

$x = \frac{8\pm\sqrt{64-12}}{6}$

$= \frac{8\pm\sqrt{52}}{6} = \frac{8\pm 2\sqrt{13}}{6}$

$\frac{4\pm\sqrt{13}}{3}$ *

Both are valid choices.

II $f(x)=\frac{ax+c}{x^2-b}$

a) $f(-x)\overset{\text{set}}{=}f(x)$

$\frac{ax+c}{x^2-b}\overset{\text{set}}{=}\frac{-ax+c}{x^2-b}$

$2ax=0$

$a=0$

$\lim_{x\to 2^+} f(x)\to\infty$ so $b=4$ Domain rest $x=2$

$f(x)=\frac{c}{x^2-4}$

$f'(x)=\frac{-2cx}{(x^2-4)^2}$

$f'(1)=\frac{-2c}{9}\overset{\text{set}}{=}-2$

$c=9$

$f(x)=\frac{9}{x^2-4}$

b) Vertical asymptotes & Domain restrictions (2 pts)

$x=-2$

$x=2$

Horizontal asymptotes when $\lim_{x\to\pm\infty} f(x)$

$\lim_{x\to\pm\infty}\frac{9}{x^2-4}\to 0$

$y=0$

c) $f'(x)=\frac{-18x}{(x^2-4)^2}$

$\frac{+\,;\,+\,0\,-\,;\,-}{-2\quad 0\quad 2}$

$f(1)=\frac{9}{-3}=-3$

$(-1,\frac{3}{32})$ Rel MIN ENDPT INC fm f'

$(0,-\frac{9}{4})$ Rel MAX f'cut

$(1,\frac{3}{32})$ Rel MIN ENDPT INC to f'

III $f(x)=\ln\left(\frac{x}{x-1}\right)$ $f(x)=\ln x-\ln(x-1)$

a) $\frac{+\,0\,-\,0\,+}{\quad 1\quad\quad 1}$ $D_g\ x>1$

$D_f\ x<0$ or $x>1$

b) They have diff domains so NOT =.

c) $f'(x)=\left(\frac{x-1}{x}\right)\frac{x-1-x}{(x-1)^2}$

$=\frac{-1}{x(x-1)}\Big|_{x=-1}=\frac{-1}{(-1)(-2)}=-\frac{1}{2}$

d) $x=\ln\frac{z}{z+1}$

$e^x=\frac{z}{z+1}$

$e^x z-e^x=z$

$z(e^x-1)=e^x$

$z=\frac{e^x}{e^x-1}=f^{-1}(x)$

e) $\frac{e^{\ln\frac{x}{x-1}}}{e^{\ln\frac{x}{x-1}}-1}$ $\ln\left(\frac{\frac{e^x}{e^x-1}}{\frac{e^x}{e^x-1}-1}\right)$

$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}=\frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}}$ $\ln\left(\frac{\frac{e^x}{e^x-1}}{e^x-e^x+1}\right)$

$\ln(e^x)$

x

IV A $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\frac{\pi}{4}}{x^2-\frac{\pi^2}{16}}$

$\lim_{x\to\frac{\pi}{4}}\frac{\sin(x-\frac{\pi}{4})}{(x-\frac{\pi}{4})}\cdot\frac{1}{x+\frac{\pi}{4}}$

$1\cdot\frac{1}{\frac{\pi}{2}}$

$\frac{2}{\pi}$

B) $\lim_{x\to 0}\frac{x-\tan x}{x-\sin x}\ \frac{0}{0}$

$\lim_{x\to 0}\frac{1-\sec^2 x}{1-\cos x}$

$\lim_{x\to 0}\frac{(\cos^2 x-1)}{\cos^2 x}$

$\lim_{x\to 0}-\frac{(\cos x+1)}{\cos^2 x}$

$-\frac{1+1}{1}=-2$

C) $\lim_{x\to 0}\frac{\sin^{-1}x}{\tan^{-1}2x}\ \frac{0}{0}$

$\lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}}{\frac{2}{1+4x^2}}=\frac{1}{2}$

$\lim e^x$

x

Advanced Placement Calculus

Name Calc Ch6 Mu

Per _____ Date 5 May

Copy original problem.

Convince *me* that **you** understand the concept*!*

No Calculators!

Chapter 6 Mechanics Exam

I Given: $f(x) = \begin{cases} -x^2 + x + 5 & \text{if } x < 0 \\ x^3 - 4x^2 + x + 5 & \text{if } x \geq 0 \end{cases}$ and $x \in [-1,1]$ (25 pts)

a) Prove the *Mean Value Theorem* applies to $f(x)$ for the given interval (ie. Prove $f(x)$ is continuous in the closed interval and differentiable in the open interval.).

b) Find all X whose existence is guaranteed by the *Mean Value Theorem* for the given interval.

II Given: $f(x) = \dfrac{ax + c}{x^2 - b}$ and $\begin{cases} i & f(x) \text{ is an even function} \\ ii & \lim\limits_{x \to 2^+} f(x) = \infty \\ iii & f'(1) = -2 \end{cases}$ and $x \in [-10,10]$ (30 pts)

a) Determine the values of a, b, and c. (Hint:: Use fact *i* to get *a*. Use fact *ii* to get *b*. Use fact *iii* to get *c*.)

b) Write the equations of any vertical or horizontal asymptotes.

c) Determine, identify, and justify the coordinates of *all* extrema.

III Given $f(x) = \ln\left(\dfrac{x}{x-1}\right)$ and $g(x) = \ln x - \ln(x-1)$ (30 pts)

a) Determine the domain for $f(x)$. Determine the domain for $g(x)$.

b) Explain how you know that $f(x)$ and $g(x)$ are not equal functions.

c) Find $f'(x)$ and then evaluate it at $x = -1$.

d) Write an expression for $f^{-1}(x)$ (ie. the inverse function).

e) Demonstrate that $f^{-1}(f(x)) = x$ and $f\left(f^{-1}(x)\right) = x$

IV Find the following limits (5 pts ea)

a) $\lim\limits_{x \to \pi/4} \dfrac{\sin\left(x - \pi/4\right)}{x^2 - \pi^2/16}$ b) $\lim\limits_{x \to 0} \dfrac{x - \tan x}{x - \sin x}$ c) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x}{\tan^{-1} 2x}$